



## Nonlinear shocklike, soliton, and periodic dust-ion-acoustic waves in Jupiter ionosphere

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### ABSTRACT

For the time being, investigation of nonlinear structures in space have become an important tool that plays a role in the flybys electronic devices, as it has a very remarkable feature to protect the electronics used in spacecrafts. The dynamics behavior of nonlinear dust-ion-acoustic shock-like, soliton, and periodic waves are investigated in a stationary positively charged dusty plasma, mobile ions and isothermal electrons those interact with streaming solar wind ions and electrons. The basic equations describing the dynamics of dust-ion-acoustic waves is reduced to one evolution equation called nonlinear Korteweg-de Vries (KdV) equation using the reductive perturbation method. Possible solutions of the KdV equation are obtained including shock-like, soliton, and periodic waves solutions using a traveling wave transformation. The effects of different plasma parameters (such as proton solar wind beam temperature, streaming velocity, and ion masses as well as Jupiter positive ions temperature and mass) on the behavior of the shocklike, soliton, and periodic waves have been examined.

### Key Words:

dust-ion-acoustic waves; Jupiter ionosphere; shocklike, soliton, and periodic waves.

### 1. INTRODUCTION

It is well known that dust particles are common in the universe and they represent much of the solid matter in it. Dust particles often contaminate fully ionized or partially ionized gases and form so-called dusty plasma, which occur frequently in nature. In astrophysics, in the early 1930s, dust was shown to be present in the interstellar clouds where it appears as a selective absorption of stellar radiation interstellar reddening [1]. The dust particles are of micrometer or submicrometer size, and the mass of the dust particles is very large. Due to the presence of such heavy particles, the plasma normal mode could be modified. In particular, the ion-acoustic waves are one of the modified normal modes, which

are called dust-ion-acoustic (DIA) waves. Shukla and Silin were the first to report theoretically the existence of DIA waves in unmagnetized dusty plasma [2]. The DIA waves have been experimentally observed in laboratory by Barkan et al [3]. It was noted that in studying collective effects involving charged dust particles in dusty plasmas, one generally assumes that the dust grains behave like point charges [4].

The solution wave in dusty plasma can be formed by different means. These are not necessarily restricted to the mode excitation due to instabilities, or an external forcing, but can also be a regular collective process. The anomalous dissipation in dusty plasma, which originates from the dust particles charging process, makes possible the existence of a new kind of shocks or shock like waves due to dissipation [5, 6]. These nonlinear waves have attracted much regard because of their importance to interpret many basic phenomena in physics. A good expectation of nonlinear system is the plasma physics since the nonlinearity of the plasma is raised for many reasons due to unforeseen circumstances in the particles dynamics. Basically, the nonlinear structures, which represent the plasma states far from thermodynamic equilibrium, are either spontaneously created in laboratory and space plasma or externally launched in laboratory under certain conditions [7, 8]. On the other hand, in laboratory experiments the scientists may control the conditions to produce certain wave. However, in the natural universe the perturbations appear and grow with no expecting what kind of wave evolution [9, 10].

Different kind of nonlinear waves were investigated in plasma including solitary waves, double layers or sometimes called shocklike, shock waves, rogue waves, etc. (see e.g. Refs. [11, 12, 13, 14]). These investigations based on solving the nonlinear partial differential equations, which describe the plasma systems, by various techniques such as perturbation methods. The obtained equations are solved either analytically or numerically to describe the physical nonlinear phenomena. One of the interesting nonlinear phenomena in science (and recently in plasma physics) is the cnoidal or periodic waves.

The motivation of the present work is to investigate the DIA waves that could produce in Jupiter ionosphere due to the interaction between the streaming solar wind with the Jupiter ionosphere constituents. This interaction may give rise to generate different nonlinear modes [14, 15, 16]. The outline of the manuscript is as follow: In Sec. 2, we present the basic equations describing the dynamics of the nonlinear DIW waves. We use the reductive perturbation method to derive an evolution equation. The solutions of the latter are obtained. In Sec. 3, the numerical results are presented. Finally, the results are summarized in Sec. 4.

## 2. Theoretical model and evolution equation

Let us consider five components unmagnetized collisionless dusty plasma composed of stationary positively charged dust grains, fluid positive ions, and proton streaming from the solar wind, as well as isothermal electrons from solar wind and in Jupiter itself. The nonlinear fluid dynamics are governed by set of the normalized basic fluid equations as: For the fluid positive ions

$$\frac{\partial n_i}{\partial t} + \frac{\partial}{\partial x}(n_i u_i) = 0, \quad (1)$$

$$\frac{\partial u_i}{\partial t} + u_i \frac{\partial u_i}{\partial x} + \sigma_1 n_i \frac{\partial n_i}{\partial x} + \frac{\partial \phi}{\partial x} = 0, \quad (2)$$

and the proton beam fluid equations are expressed as

$$\frac{\partial n_p}{\partial t} + \frac{\partial}{\partial x}(n_p u_p) = 0, \quad (3)$$

$$\frac{\partial u_p}{\partial t} + u_p \frac{\partial u_p}{\partial x} + \frac{\sigma_2}{Q} n_p \frac{\partial n_p}{\partial x} + \frac{1}{Q} \frac{\partial \varphi}{\partial x} = 0, \quad (4)$$

The isothermal electrons in the Jupiter and solar wind are expressed, respectively, as

$$n_e = \exp(\varphi), \quad (5)$$

and

$$n_{es} = \exp(\sigma_{es}\varphi). \quad (6)$$

Equations (1)–(6) are closed by the Poisson equation

$$\frac{\partial^2 \varphi}{\partial x^2} + n_i + \mu_p n_p - \mu_e n_e - \mu_{es} n_{es} + \alpha = 0, \quad (7)$$

where  $\mu_p = \frac{n_{p0}}{n_{i0}}$ ,  $\mu_e = \frac{n_{e0}}{n_{i0}}$ ,  $\mu_{es} = \frac{n_{es0}}{n_{i0}}$ ,  $\alpha = \frac{Z_{d0} n_{d0}}{n_{i0}}$  are the ratios of unperturbed charges densities-to-ion density.  $u_i$ , and  $u_p$  are the fluid velocities of the positive ion and proton beam, respectively, which are normalized by the ion-acoustic speed  $C_i = (k_B T_e / m_i)^{1/2}$ ,  $\varphi$  is the electrostatic potential is normalized by  $k_B T_e / e$ . The space coordinate  $x$  and the time  $t$  are normalized by the Debye length  $\lambda_{Di} = (k_B T_e / 4\pi n_{i0} e^2)^{1/2}$  and the inverse of ion plasma frequency  $\omega_{pi}^{-1} = (m_d / 4\pi n_{i0} e^2)^{1/2}$ .  $\sigma_1 = (3T_i / T_e)$ ,  $\sigma_2 = (3T_p / T_e)$ ,  $\sigma_{es} = T_{es} / T_e$ ,  $Q = m_p / m_i$ , but for  $m_p = m_i$  which is the case at hand then  $Q = 1$ .

The charge-neutrality condition in the plasma is always maintained through the relation

$$n_{p0} - n_{e0} + n_{i0} + n_{es0} + Z_{d0} n_{d0} = 0, \quad (8)$$

where  $n_{p0}$ ,  $n_{e0}$ ,  $n_{i0}$ ,  $n_{es0}$ , and  $n_{d0}$  are the initial/unperturbed densities of the charged particle.

To derive an evolution equation describing the system, we employ a reductive perturbation method. According to this method, we introduce the stretched space–time coordinates [15, 16]:

$$\xi = \varepsilon^{1/2}(x - V_p t) \text{ and } \tau = \varepsilon^{3/2} t, \quad (9)$$

where  $\varepsilon$  is a small parameter (i.e.  $0 < \varepsilon \ll 1$ ) and  $V_p$  is the phase velocity of the propagating electrostatic wave. The normalized dependent variables are expressed as

$$n_p = 1 + \varepsilon n_{p1} + \varepsilon^2 n_{p2} + \varepsilon^3 n_{p3} + \dots, n_i = 1 + \varepsilon n_{i1} + \varepsilon^2 n_{i2} + \varepsilon^3 n_{i3} + \dots, \quad (10)$$

$$u_p = u_{p0} + \varepsilon u_{p1} + \varepsilon^2 u_{p2} + \varepsilon^3 u_{p3} + \dots, u_i = \varepsilon u_{i1} + \varepsilon^2 u_{i2} + \varepsilon^3 u_{i3} + \dots, \quad (11)$$

$$n_e = 1 + \varepsilon n_{e1} + \varepsilon^2 n_{e2} + \varepsilon^3 n_{e3} + \dots, n_{es} = 1 + \varepsilon n_{es1} + \varepsilon^2 n_{es2} + \varepsilon^3 n_{es3} + \dots, \quad (12)$$

and

$$\varphi = \varepsilon \varphi_1 + \varepsilon^2 \varphi_2 + \varepsilon^3 \varphi_3 + \dots \quad (13)$$

Applying the relations (9)–(13), to the basic fluid equations (1)–(7), and following the usual procedure of the reductive perturbation theory, the lowest-order of positive ions gives

$$\frac{\partial n_{i1}}{\partial \xi} = \frac{1}{V_p^2 - \sigma_1} \frac{\partial \varphi_{i1}}{\partial \xi}, \quad \frac{\partial u_{i1}}{\partial \xi} = \frac{V_p}{V_p^2 - \sigma_1} \frac{\partial \varphi_{i1}}{\partial \xi}, \quad (14)$$

while the proton beam yields

$$\frac{\partial n_{p1}}{\partial \xi} = \frac{1}{(u_{p0} - V_p)^2 - \sigma_2} \frac{\partial \varphi_{i1}}{\partial \xi}, \quad \frac{\partial u_{p1}}{\partial \xi} = \frac{u_{p0} - V_p}{(u_{p0} - V_p)^2 - \sigma_2} \frac{\partial \varphi_{i1}}{\partial \xi}. \quad (15)$$

The electrons of Jupiter and solar wind have the relations

$$n_{e1} = \varphi_1, \quad n_{es1} = \sigma_{es} \varphi_1, \quad (16)$$

and the Poisson equation gives the following compatibility condition

$$\frac{1}{V_p^2 - \sigma_1} + \frac{\mu_p}{(u_{p0} - V_p)^2 - \sigma_2} - \mu_e - \mu_{es} \sigma_{es} = 0. \quad (17)$$

Take into account the next-order in  $\varepsilon$ , we obtain a system of equations in the second-order perturbed quantities. Solving this system with the aid of Eqs. (14)–(17), we finally obtain an evolution equation in the form of Korteweg-de Vries (KdV) equation

$$\frac{\partial \varphi_1}{\partial \tau} + A \varphi_1 \frac{\partial \varphi_1}{\partial \xi} + B \frac{\partial^3 \varphi_1}{\partial \xi^3} = 0. \quad (18)$$

where the second and third terms in last equation represent the nonlinear and dispersion terms, respectively. Here the coefficients  $A$  and  $B$  are given by

$$A = B \left( \frac{\mu_p (3(u_{p0} - V_p)^2 + \sigma_2)}{((u_{p0} - V_p)^2 - \sigma_2)^3} + \frac{3V_p^2 + \sigma_1}{(V_p^2 - \sigma_1)^3} - \mu_e - \mu_{es} \sigma_{es}^2 \right).$$

and

$$B = \left( \frac{2V_p}{(V_p^2 - \sigma_1)^2} - \frac{2\mu_p(u_{p0} - V_p)}{((u_{p0} - V_p)^2 - \sigma_2)^2} \right)^{-1}.$$

To obtain the possible solutions of Eq. (18), we assume that a traveling wave solution  $\varphi_1(\tau, \xi) \equiv F(y)$ , where  $y = \xi - C_s \tau$  and  $C_s$  is the velocity of the nonlinear structure moving with the frame. It is interesting to mention here the solution of Eq. (18) is obtained in stationary form since we are interested to investigate the localized solutions only. Using the last transformation into Eq. (18) and integrating once, we obtain an ordinary differential equation in the form

$$\frac{d^2 F}{dy^2} + B_1 F^2 + B_2 F = 0. \quad (19)$$

Here,  $B_1 = A/(2B)$ ,  $B_2 = (-C_s)/B$ . The next crucial step is that the solution of Eq. (19) we are looking for is expressed in the form [19, 20]

$$F(y) = \sum_{i=0}^n a_i \left( \frac{G'}{G} \right)^i + \sum_{i=1}^n b_i \left( \psi_i(y) + \frac{G'}{G} \right)^{-i}, \quad (20)$$

where  $\psi_i(y)$  are arbitrary functions in  $y$ , and  $a_i$  and  $b_i$  are constants dependent on the physical parameters of the system. The positive integer  $n$  can be determined by balancing the highest-order derivatives with the highest-order nonlinear terms of Eq. (19). This gives us  $n = 2$ . Substituting Eq. (20) into Eq. (19) and collecting a power derivative of  $G$ , we get the constants in terms of the physical parameters as

$$\begin{aligned} a_0 &= \frac{\sqrt{5(B_2^2)-5B_2}}{10B_1}, a_1 = \frac{-(B_2^2)^{1/4}}{5^{1/4}B_1}, a_2 = \frac{-1}{B_1}, \\ b_1 &= \frac{-(B_2^2-4B_1B_3)^{3/4}}{2 \times 5^{3/4}B_1}, b_2 = \frac{-B_2^2}{20B_1}. \end{aligned} \quad (21)$$

$$\begin{aligned} a_0 &= \frac{-\sqrt{5(B_2^2)-5B_2}}{10B_1}, a_1 = \frac{i(B_2^2)^{1/4}}{5^{1/4}B_1}, a_2 = \frac{-1}{B_1}, \\ b_1 &= \frac{i(B_2^2)^{3/4}}{2 \times 5^{3/4}B_1}, b_2 = \frac{-B_2^2}{20B_1}. \end{aligned} \quad (22)$$

$$\begin{aligned} a_0 &= \frac{5B_2-\sqrt{5(B_2^2)}}{10B_1}, a_1 = \frac{-i(B_2^2)^{1/4}}{5^{1/4}B_1}, a_2 = \frac{-1}{B_1}, \\ b_1 &= \frac{i(B_2^2)^{3/4}}{2 \times 5^{3/4}B_1}, b_2 = \frac{-B_2^2}{20B_1}. \end{aligned} \quad (23)$$

$$\begin{aligned} a_0 &= \frac{-5B_2+\sqrt{5(B_2^2-4B_1B_3)}}{10B_1}, a_1 = \frac{(B_2^2-4B_1B_3)^{1/4}}{5^{1/4}B_1}, a_2 = \frac{-1}{B_1}, \\ b_1 &= \frac{(B_2^2)^{3/4}}{2 \times 5^{3/4}B_1}, b_2 = \frac{-B_2^2}{20B_1}. \end{aligned} \quad (24)$$

Due to this method, the function  $G(y)$  have to satisfy the Riccati equation

$$\frac{d^2G}{dy^2} + \gamma_1 \frac{dG}{dy} + \gamma_2 G = 0, \quad (25)$$

where  $\gamma_1$  and  $\gamma_2$  are constants to be determined later. The possible solutions of Eq. (25) give

$$G(y) = \exp\left(\frac{-\gamma_1}{2}y\right) (r_1 \sinh[\theta_1 y] + r_2 \cosh[\theta_1 y]), \quad (23)$$

with  $r_1 > r_2$ ,  $(\gamma_1^2 - 4\gamma_2^2)^{1/2} > 0$ , and  $\theta_1 = (\gamma_1^2 - 4\gamma_2^2)^{1/2}$ ,

$$G(y) = \exp\left(\frac{-\gamma_1}{2}y\right) (r_3 \sin[\theta_2 y] + r_4 \cos[\theta_2 y]), \quad (24)$$

with  $r_3 < r_4$ ,  $(\gamma_1^2 - 4\gamma_2^2)^{1/2} < 0$ , and  $\theta_2 = (\gamma_1^2 - 4\gamma_2^2)^{1/2}/i$ ,

$$G(y) = \exp\left(\frac{-\gamma_1}{2}y\right) (r_5 + r_6), \quad (25)$$

with  $(\gamma_1^2 - 4\gamma_2^2)^{1/2} = 0$ ,

Employing an expression of  $G(y)$  from the solutions (22)–(25) with the constants (18)–(21) into the general solution (17), we obtain different solutions of Eq. (16). Solution (23) expresses three different wave profiles, namely, soliton, explosive (blowup), and shocklike (double layer) solutions based on the arbitrary constants, while solution (24) gives a periodic wave structure only.

### 3. Results and Discussion

It is clear from the analytical expressions presented in Section 2 that the properties of the nonlinear waves depend on the intrinsic plasma parameters of the Jupiter. On the other hand, we will analyze, the parametric dependence of the pulse profile on the plasma parameters of the Jupiter plasma system. These plasma parameters were taken from [12, 13]

The wave solution describes the shocklike wave is represented by Eq. (18). It is interesting to mention here that we shall focus our efforts to investigate the effects of proton beam parameters namely  $\sigma_2$ ,  $u_{p0}$ , and  $\mu_p$ , as well as the density of stationary dust grains  $\alpha$  on the shocklike profile as depicted in Figure 1. When the proton beam temperature increases the amplitude of the shocklike wave decreases. While for denser and faster proton beam as well as high concentration dust grains, the pulse profile becomes taller and then carry more energy to influence on any electronic devices flyby near the atmosphere.

A second kind of nonlinear waves is the soliton waves, which occur when a critical balance dispersion and nonlinearity is presented. This balance allows to propagate a localized pulses in the form of soliton wave. The latter can exist for long time without changing in its profile, so without loss of generality we can consider it a stationary wave. No doubt that the soliton can transfer a reasonable amount of energy in the plasma system without suffering of dispersion and dissipation. Here, our solution (18) can express sufficiently about propagation of soliton wave. Figure 2 shows the effect of various plasma parameters on the profile of soliton wave. It is seen that  $\sigma_2$ ,  $u_{p0}$ ,  $\mu_p$ , and  $\alpha$  lead to make the soliton wave taller and wider. Because of the soliton wave theory, taller soliton will be faster. Therefore, increasing the plasma parameters makes the soliton swifter and energetic.

Another type of nonlinear waves that may exist at Jupiter atmosphere is the periodic waves. Indeed, this kind of waves carry the energy for long scale since it repeats itself continuously. Thus, it is of our interest to examine the wave profile especially the amplitude to understand which physical parameter can enhance the wave energy. From figure 3, it is obvious that both of  $\sigma_2$  and  $\mu_p$  makes the periodic wave taller, i.e. the wave gains higher energy, whilst  $u_{p0}$  and  $\alpha$  shrinks the amplitude profile and leads to reduce the wave energy in the Jupiter atmosphere.

### 4. Summary

In this paper, the different nonlinear dust-ion-acoustic waves have been investigated in collisionless, unmagnetized plasma, consisting of stationary positively charged dust grains, fluid ions in Jupiter and solar wind, as well as isothermal electrons in Jupiter and solar wind. The reductive perturbation method is used to reduce the basic set of fluid equations to one evolution equation called KdV equation. The dynamics behavior of nonlinear dust-ion-acoustic waves can occur in the form of shocklike, soliton, and periodic waves. The analytical solutions describing these waves are presented. The effects of solar wind parameters (density, temperature and streaming speed), as well as dust density are examined on the pulses profile. It is found that When the proton beam temperature increases the amplitude of the shocklike wave decreases. While for denser and faster proton beam as well as high concentration dust grains, the pulse profile becomes taller and carry more energy. Furthermore, all the physical parameters

are enhanced the soliton amplitude that lead to the waves transfer additional energy. While for periodic waves, the higher temperature and density makes the periodic wave taller and thus the wave gains higher energy. The proton streaming speed and dust density shrinks the amplitude profile and leads to reduce the wave energy in the Jupiter atmosphere.

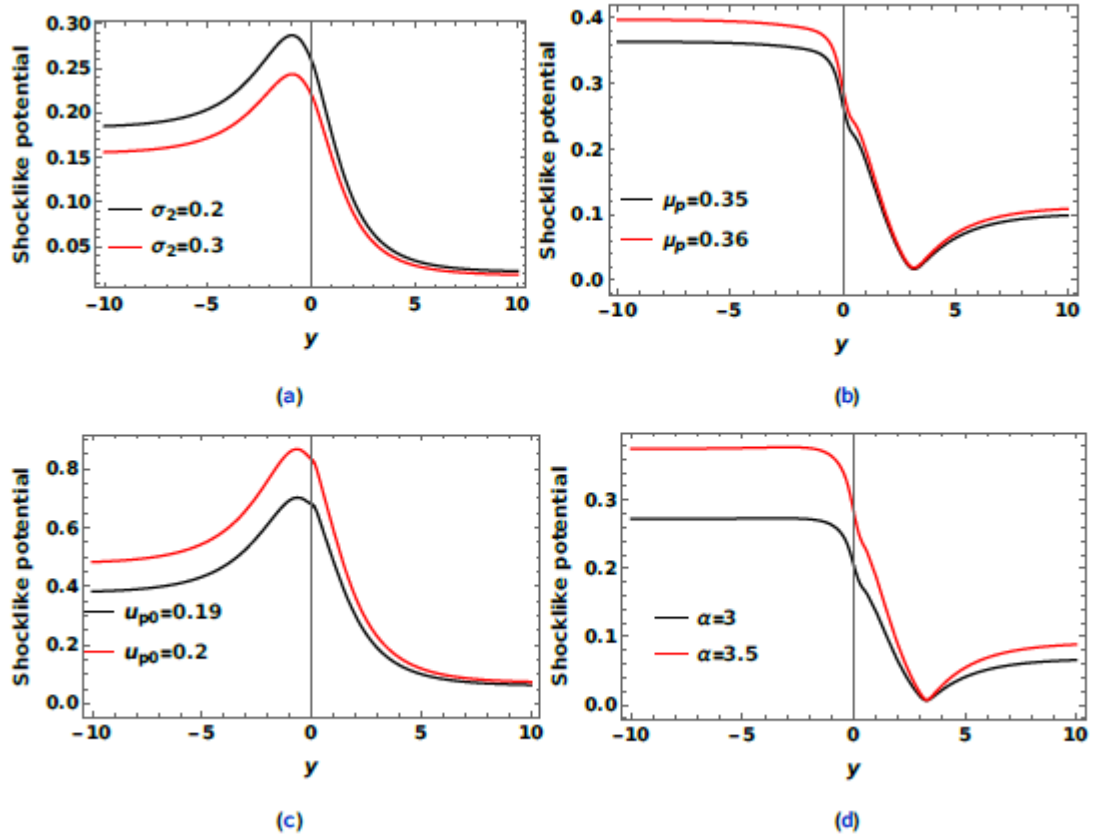


Figure 1: (Color online) The normalized shocklike wave potential for different values of  $\sigma_2$ ,  $\mu_p$ ,  $u_{p0}$ , and  $\alpha$  where the plasma parameters are  $\mu_{es} = 0.43$ ,  $\sigma_{es} = 0.2$ ,  $\sigma_1 = 0.1$ .

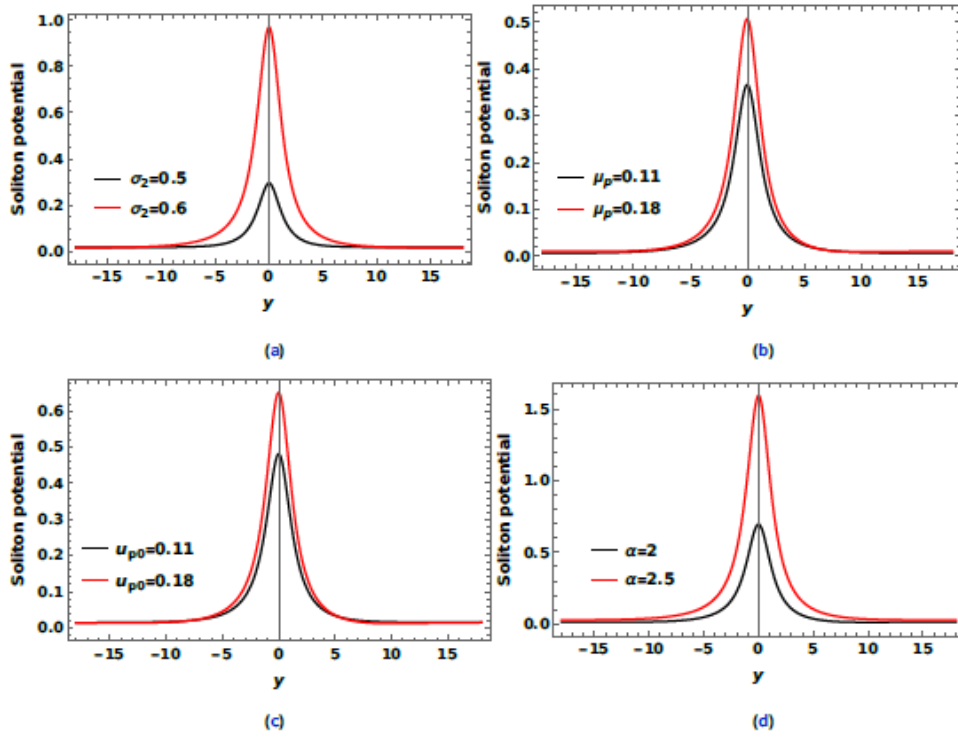


Figure 2: (Color online) The normalized soliton wave potential for different values of  $\sigma_2$ ,  $\mu_p$ ,  $u_{p0}$ , and  $\alpha$  where the plasma parameters are  $\mu_{es} = 0.43$ ,  $\sigma_{es} = 0.18$ ,  $\sigma_1 = 0.61$ .

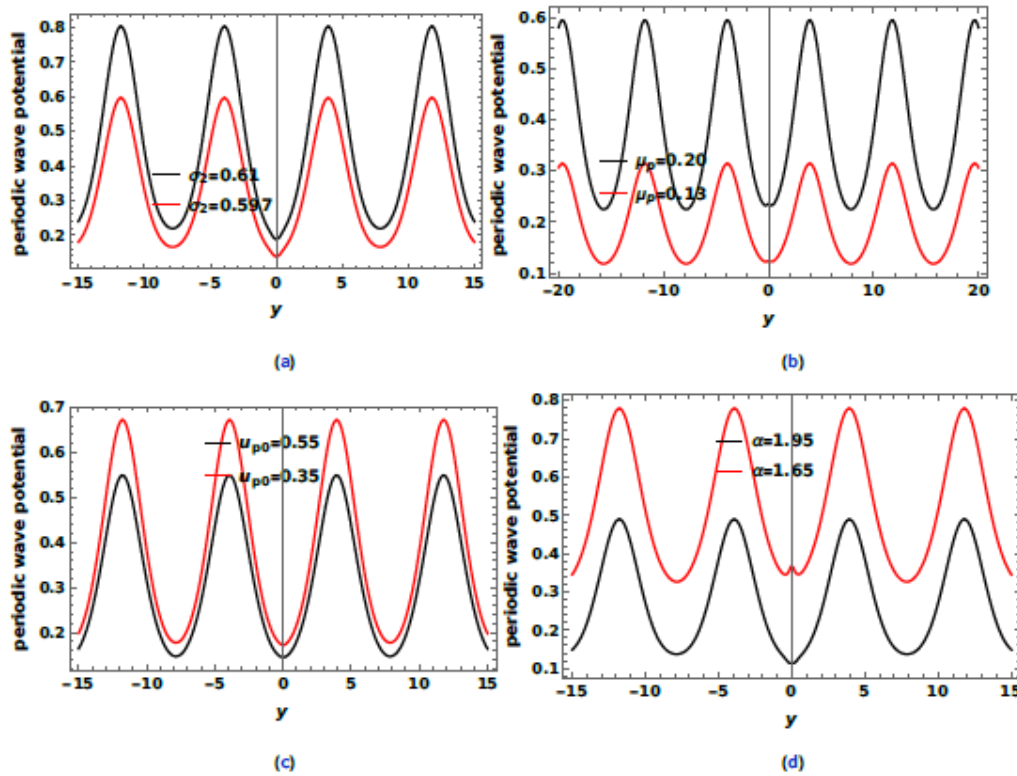


Figure 3: (Color online) The normalized periodical wave potential for different values of  $\sigma_2$ ,  $\mu_p$ ,  $u_{p0}$ , and  $\alpha$  where the plasma parameters are  $\mu_{es} = 0.43$ ,  $\sigma_{es} = 0.2$ ,  $\sigma_1 = 0.6$ .



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