



## Dust-ion-acoustic solitary, kink, and periodic waves on the lunar dark side

Nora A. Omar and <sup>1,\*</sup>, R. E. Tolba <sup>2</sup>

<sup>1</sup>Department of mathematics and computer science, Faculty of science, Port Said university

<sup>2</sup> Centre for Theoretical Physics, The British University in Egypt (BUE), El-Shorouk City, Cairo, Egypt

\*Corresponding author: [noraalaa19862013@gmail.com](mailto:noraalaa19862013@gmail.com)

### ABSTRACT

Plasma hydrodynamic equations are used to investigate the nonlinear dust-ion-acoustic waves on the lunar dark side that are created by plasma interaction with the Earth's magnetosphere. A plasma system is reduced to one evolution equation, called the Korteweg-de Vries equation. The latter has been solved using the new generalized (G'/G)-expansion method. Different kinds of nonlinear solutions, such as solitary, kink, and periodic solutions, are obtained. The inclusion of plasma quantities in the system significantly affects the nonlinear properties of the DIAWs. The current study holds significance in understanding how Earth's magnetosphere interacts with the lunar dark side. Understanding the dust-ion-acoustic waves on the lunar dark side is important for space physics investigations. The properties of all physical plasma parameters, as well as the nature of the planetary obstacle, determine this nonlinearity.

### Key Words:

solitary, kink, and periodic waves; lunar dark side; new generalized (G'/G)-expansion method.

### 1. INTRODUCTION

Dusty plasma, also known as complex plasma, is a distinct type of plasma containing solid particles or grains suspended within ionized gas. These grains can be charged either positively or negatively depending on the balance between the interactions of electrons and ions with the surface of the grains [1, 2]. The presence of dust grains significantly changes the behavior and properties of the plasma. Grains can become highly charged due to collect or remove electrons and ions from their surfaces.. This introduces additional forces and interactions within the plasma system, leading to unique phenomena and complex dynamics [3-5]. Dusty plasmas have been observed in various astrophysical environments, including cometary tails. When a comet approaches the Sun, its icy nucleus sublimates, releasing gas and dust particles into space [6, 7]. The interaction between the released material and a solar wind, which is a plasmas of charged particles emitted by the Sun, forms a dusty plasmas environment [8]. The dust grains in cometary tails can become charged and interact with the surrounding plasma, creating fascinating structures and phenomena [9, 10].

The presence of dust grains in a plasmas introduces new dynamics and forces that are not present in ordinary electron- ion plasmas. Due to their relatively high mass and charge, dust grains significantly

affect the collective behavior of the plasma. Two important forces that come into play in dusty plasmas are gravitational and polarization forces. Gravitational forces play a crucial role in dusty plasmas because the dust grains have mass [11]. The gravitational force between dust grains can cause them to aggregate and form larger structures such as clusters or chains. These structures can influence the overall behavior of the plasma, affecting phenomena like wave propagation and particle transport.

Polarization forces arise due to the presence of charged dust grains in the dust plasma. Thus when a charged dust grain is immersed in the plasma, it creates an electric field around itself, known as a Debye sheath [12-13]. As a result of the presence of an electric field, this can lead to the polarization of nearby particles in the plasma, creating an attractive or repulsive force between the charged dust grains and the surrounding plasma. The deformation of the Debye sheath around the dust grains is responsible for the polarization forces, which can significantly influence the motion and interactions of the dust grains [1, 14]. Studying the characteristics of dusty plasmas, including the gravitational and polarization forces, is important for understanding various astrophysical phenomena and for practical applications in plasma technology [15, 16]. Researchers investigate the behavior of dusty plasmas in laboratory experiments and through theoretical modeling to gain insights into their complex dynamics and to develop a better understanding of the underlying physical processes [4, 5].

The concept of trapping in plasma has been an important area of research in plasma physics. Trapping refers to the confinement of charged particles, such as electrons or ions, within a specific region or potential well within a plasma. One of the earliest instances of trapping in plasma was observed in the field of magnetic fusion research [17].

Extensive research has been conducted on the study of dust -acoustic nonlinear waves, in the presence of particles trapped in dust plasmas. Dust-acoustic nonlinear waves are defined as those low-frequency electrostatic waves that spread in dusty plasma, where charged dust particles interact with the surrounding plasma particles. Trapped particles, such as charged dust grains or ions, can thus significantly affect the properties and behavior of dust sound waves. Their presence introduces additional nonlinearities and can lead to complex dynamics and phenomena. Some of the key aspects that have been investigated in the study of dust-acoustic waves with trapped particles include [1, 18]. Nonlinear wave structures: Trapped particles can introduce nonlinearity into the dust-acoustic wave dynamics. Investigations have focused on nonlinear wave structures such as solitary waves, shocks, and double layers in dusty plasmas. These nonlinear structures have been studied both theoretically and through computer simulations [19, 20]. Wave dispersion and damping: Trapped particles can modify the dispersion characteristics and damping of dust-acoustic waves. The presence of trapped particles can result in modified dispersion relations, which affect the wave propagation properties. Damping mechanisms, such as collisional and Landau damping, have also been studied in the presence of trapped particles [21,22]. Instabilities: Trapped particles can give rise to various instabilities in dusty plasmas. For example, the presence of trapped ions can lead to the formation of ion-acoustic instabilities, which can affect the properties of dust-acoustic waves. Instabilities associated with trapped dust particles, such as dust-ion-acoustic instabilities, have also been investigated [23,24]. Nonlinear wave-particle interactions: Trapped particles can interact with dust-acoustic waves through nonlinear mechanisms. These interactions can lead to particle trapping, energy transfer between waves and particles, and the formation of coherent structures. Understanding these wave-particle interactions is crucial for studying the dynamics of dusty plasmas [25].

This frequency shift can lead to changes in the solitons energy through its interaction with dispersion properties of the medium. Additionally, the polarization force can modify the effective refractive index experienced by the soliton, affecting its velocity and propagation characteristics. Furthermore, the interaction between the polarization force and the soliton can result in nonlinear phase modulation, leading to frequency chirping or spectral broadening of the soliton. These effects can influence the energy distribution of the soliton and may lead to energy transfer between the dark and antidark components [1, 46]. The aims or purpose of this paper are as follows: (1) find the wave solutions of solitary waves, waves and kink waves is possible in case of a negatively charged dusty plasma, and (2) Describe the nonlinear waves of dust ions that can occur in the plasma system in the Earth's magnetosphere. In Section 2, the basic hydrodynamic equations for the dusty plasma system are presented. The KdV equation was obtained using the reduction perturbation method. The penultimate section 3 contains the numerical

analysis in describing the results and discussion. Finally, the results we obtained will be summarized in Section 4.

### 2. GOVERNING EQUATIONS AND DERIVATION OF EVOLUTION EQUATION

In this section, we investigate and study the influence of dust plasmas parameters on the fundamental features of fully nonlinear electrostatic nonlinear waves. For this purpose, the basic system of dusty plasmas containing the fluid dynamics equations is normalised as follows [26]:

$$\frac{\partial n_i}{\partial t} + \frac{\partial}{\partial x}(n_i u_i) = 0, \tag{1}$$

$$\left[ \frac{\partial}{\partial t} + u_i \frac{\partial}{\partial x} \right] u_i = -\frac{\partial \varphi}{\partial x} - 3\sigma_i n_i \frac{\partial n_i}{\partial x}, \tag{2}$$

$$\frac{\partial n_d}{\partial t} + \frac{\partial}{\partial x}(n_d u_d) = 0, \tag{3}$$

$$\left[ \frac{\partial}{\partial t} + u_d \frac{\partial}{\partial x} \right] u_d = \mu_d \frac{\partial \varphi}{\partial x}, \tag{4}$$

$$n_e = \exp[\varphi]. \tag{5}$$

The system of Eqs. (1) → (5) are closed by Poisson equation

$$\frac{\partial^2 \varphi}{\partial x^2} = \delta_e n_e + (1 - \delta_e) n_d - n_i. \tag{6}$$

Here  $n_{i,d}$  are the ions number density and negative dust grains density, and  $u_{i,d}$  are the ions and negative dust grains velocities.  $n_e$  it is an electrons number density and  $\varphi$  it is an electrostatic potential. The number densities  $n_{e,i,d}$  are normalized by  $n_{e0,i0,d0}$ . The space coordinate  $x$  and the time  $t$  are normalized by the Debye length  $\lambda_{Di} = (K_B T_e / 4\pi n_{i0} e^2)^{1/2}$  and the inverse of the plasma frequency  $\omega_{pi}^{-1} = (m_i / 4\pi n_{i0} e^2)^{1/2}$ , while the velocities and an electrostatic potential  $\varphi$  are normalised with respect to ion acoustic speed  $C_i = (K_B T_e / m_i)^{1/2}$  and  $K_B T_e / e$ , respectively. Here  $\sigma_i = (T_i / T_e)$ ,  $\mu_d = m_i / m_d$ , and  $\delta_e = n_{e0} / n_{i0}$ , where  $T_i$  is a positive ions temperature, ( $T_e$ ) is the electrons temperature. To examine the ion-acoustic waves, the independent variables are stretched as [26].

$$\xi = \varepsilon^{1/2} (x - V_p t) \text{ and } \tau = \varepsilon^{3/2} t \tag{7}$$

Where  $0 < \varepsilon < 1$  is a small (real) parameter and  $V_p$  is the phase velocity to be determined later. The dependent variables are expanded as:

$$\begin{aligned} n_i &= 1 + \varepsilon n_{i1} + \varepsilon^2 n_{i2} + \varepsilon^3 n_{i3} + \dots, \\ n_d &= 1 + \varepsilon n_{d1} + \varepsilon^2 n_{d2} + \varepsilon^3 n_{d3} + \dots, \\ n_e &= 1 + \varepsilon n_{e1} + \varepsilon^2 n_{e2} + \varepsilon^3 n_{e3} + \dots, \\ u_i &= \varepsilon u_{i1} + \varepsilon^2 u_{i2} + \varepsilon^3 u_{i3} + \dots, \\ u_d &= \varepsilon u_{d1} + \varepsilon^2 u_{d2} + \varepsilon^3 u_{d3}, \dots, \\ \varphi &= \varepsilon \varphi_1 + \varepsilon^2 \varphi_2 + \varepsilon^3 \varphi_3 + \dots, \end{aligned} \tag{8}$$

Substituting (7) and (8) into a basic set of dynamics equations (1) – (6), from the lowest order in  $\xi$ , we get:

$$\begin{aligned} V_p \frac{dn_{i1}}{d\xi} &= \frac{du_{i1}}{d\xi} \\ V_p \frac{dn_{d1}}{d\xi} &= \frac{du_{d1}}{d\xi} \\ -V_p \frac{du_{i1}}{d\xi} + \frac{d\varphi_1}{d\xi} + 3\sigma_i \frac{dn_{i1}}{d\xi} &= 0 \\ -V_p \frac{du_{d1}}{d\xi} - \mu_d \frac{d\varphi_1}{d\xi} &= 0 \end{aligned} \tag{9-a}$$

By Solving (9-a), we get:

$$n_{i1} = \frac{\varphi_1}{V_p^2 - 3\sigma_i}$$

$$u_{i1} = \frac{V_p \varphi_1}{V_p^2 - 3\sigma_i}$$

$$n_{d1} = \frac{-\mu_b \varphi_1}{V_p^2} \tag{9-b}$$

$$u_{d1} = \frac{-\mu_b \varphi_1}{V_p}$$

$$n_{e1} = \varphi_1$$

$$V_p = \sqrt{\frac{1 + \mu_b - \delta_e \mu_b + 3\delta_e \sigma_i + \sqrt{12(-1 + \delta_e)\delta_e \mu_b \sigma_i + (1 + \mu_b - \delta_e \mu_b + 3\delta_e \sigma_i)^2}}{\delta_e \sqrt{2}}}$$

The next-order of the perturbation gives a system of differential equation, solving it with the aid of equations (1) – (6) we obtain the evolution equation (KdV equation) as:

$$\frac{\partial \varphi_1}{\partial \tau} + B\varphi_1 \frac{\partial \varphi_1}{\partial \xi} + A \frac{\partial^3 \varphi_1}{\partial \xi^3} = 0 \tag{10}$$

where,

$$A = \frac{V_p^3}{2 \left( \mu_d - \delta_e \mu_d + \frac{V_p^4}{V_p^2 - 3\sigma_i} \right)},$$

and

$$B = A \left( -\delta_e + \frac{3(\delta_e - 1)\mu_d^2}{V_p^4} + \frac{3(V_p^2 + \sigma_i)}{(V_p^2 - \sigma_i)^3} \right).$$

### 3. SOLUTIONS OF KDV EQUATION

Introducing a class of solutions of nonlinear evolution equations obtained by the new generalizer

$(G'/G) -$  Expansions [27].

We applying a new generalized  $(G'/G) -$  expansion methods for solve analytical solutions of the (Kdv Eq. 10).

We can use  $\varphi(\tau, \xi) = \varphi(X)$ , where  $X = \xi - M\tau$ , such that  $M$  is the nonlinear acoustic wave speed, and then Eq. (10) can be transform to ordinary differential Eq. [27]

$$A \frac{d^3 \varphi}{dX^3} + B\varphi \frac{d\varphi}{dX} - M \frac{d\varphi}{dX} = 0, \tag{11}$$

$\varphi \equiv \varphi_1$ , for simplicity. Now, by integration we get:

$$\frac{d^2 \varphi}{dX^2} + F_2 \varphi^2 + F_1 \varphi = 0, \tag{12}$$

where  $F_2 = B/(2A)$ ,  $F_1 = -M/A$ . We now assume that Eq. (12) has the general solution of the model

$$\varphi(X) = \sum_{l=0}^n h_l \left( \frac{G'}{G} \right)^l + \sum_{l=1}^n b_l \left( \psi_l + \frac{G'}{G} \right)^{-l} \tag{13}$$

such that  $\psi_l$  is an arbitrary function in  $X$  not equal to be zero. The condition  $G \equiv G(X)$  satisfy the Riccati equation

$$\frac{d^2 G}{dX^2} + \alpha_1 \frac{dG}{dX} + \alpha_2 G = 0, \tag{14}$$

where  $h_l, b_l, \alpha_1$  and  $\alpha_2$  are constants determined latter.

The positive real number  $n > 0$  can be determined by balancing the higher-order nonlinear terms with the higher-order derivatives that appear in the eq. (10) or in eq. (12). More precisely,  $n = 2$  we can substituting from eq. (13) into eq.(12), and collecting on power derivative of  $G$  we obtain values of the constants by the physical plasmas parameters as:

$$h_0 = \frac{(-5 + \sqrt{5})F_1}{10F_2}, h_1 = \frac{-\sqrt{F_1}}{F_2(5)^{1/4}}, h_2 = \frac{-1}{F_2}, b_1 = \frac{F_1^{3/2}}{2F_2(5)^{3/4}}, b_2 = \frac{F_1^2}{20F_2}, \tag{15 - a}$$

$$h_0 = \frac{(-5 - \sqrt{5})F_1}{10F_2}, h_1 = \frac{i\sqrt{F_1}}{F_2(5)^{1/4}}, h_2 = \frac{-1}{F_2}, b_1 = \frac{-iF_1^{3/2}}{2F_2(5)^{3/4}}, b_2 = \frac{-F_1^2}{20F_2}, \tag{15 - b}$$

$$h_0 = \frac{(-5 - \sqrt{5})F_1}{10F_2}, h_1 = \frac{-i\sqrt{F_1}}{F_2(5)^{1/4}}, h_2 = \frac{-1}{F_2}, b_1 = \frac{iF_1^{3/2}}{2F_2(5)^{3/4}}, b_2 = \frac{-F_1^2}{20F_2}, \tag{15 - c}$$

$$h_0 = \frac{(-5 + \sqrt{5})F_1}{10F_2}, h_1 = \frac{\sqrt{F_1}}{F_2(5)^{1/4}}, h_2 = \frac{-1}{F_2}, b_1 = \frac{F_1^{3/2}}{2F_2(5)^{3/4}}, b_2 = \frac{-F_1^2}{20F_2}, \tag{15 - d}$$

Now, we can solve the ordinary differential eq. (14)

1- If  $(\alpha_1^2 - 4\alpha_2^2)^{1/2} > 0$  ,  $\theta_1 = (\alpha_1^2 - 4\alpha_2^2)^{1/2}$   
 $\therefore G(X) = \exp\left(\frac{-\alpha_1}{2}X\right) (c_1 \sinh(\theta_1 X) + c_2 \cosh(\theta_1 X))$  ,  $c_1 > c_2$  (16)

2- If  $(\alpha_1^2 - 4\alpha_2^2)^{1/2} < 0$  ,  $\theta_2 = (\alpha_1^2 - 4\alpha_2^2)^{1/2}/i$   
 $\therefore G(X) = \exp\left(\frac{-\alpha_1}{2}X\right) (c_3 \sin(\theta_2 X) + c_4 \cos(\theta_2 X))$  ,  $c_3 < c_4$  (17)

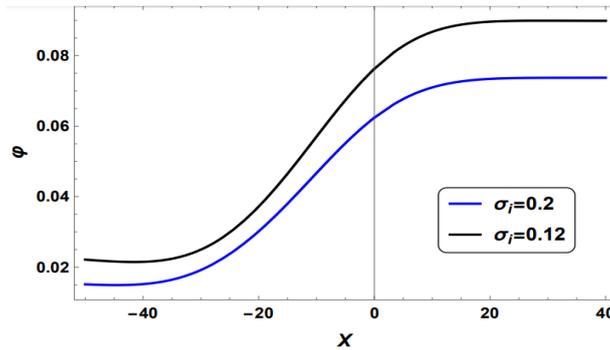
3- If  $(\alpha_1^2 - 4\alpha_2^2)^{1/2} = 0$  ,  
 $G(X) = \exp\left(\frac{-\alpha_1}{2}X\right) (c_5 + Xc_6)$  , (18)

Solution (16) with the aid of constants (15), it may be useful to explain some physical plasmas phenomena in the solar wind, this solution give me the Explosive pulse, shock like wave solution in the solar wind and solitary waves [14], solutions (17) gives periodic waves for Kdv Eq. (10).

#### 4. DISCUSSION

In order to study and keep the analytical results physically meaningful, we focus our attention on investigating the properties of nonlinear wave potentials in the lunar dark side dust plasma caused by the dynamical interaction with the Earth's magnetosphere. Establishing capabilities and identifying the physical mechanisms behind their disposal is of effective importance to avoid any problems or obstacles to the electronics of satellites and spacecraft. It is known that ions and electrons play important roles in plasma physics, as plasma behavior is differentially and effectively affected by changes in temperature. In plasmas, of charged particles, therefore, the temperature of all both the ions and electrons can affect the general properties of the plasma. That is, when the temperature of positive ions decreases, this usually leads to a decrease in their kinetic energy. As a result, the ions move more slowly and have lower average velocities. This decrease in ion temperature leads to an increase in a amplitude of the nonlinear shock-like waves. On the other hand, we find that the behavior of electrons is deeply related to wave phenomena in plasma. The temperature of electrons directly affects their thermal motion and average kinetic energy. Therefore, when the temperature of electrons increases, it leads to higher electron velocities and more active electronic interactions within the plasma environment. These energetic interactions can therefore contribute to the generation and propagation of shock-like waves. Therefore, it is more accurate to say

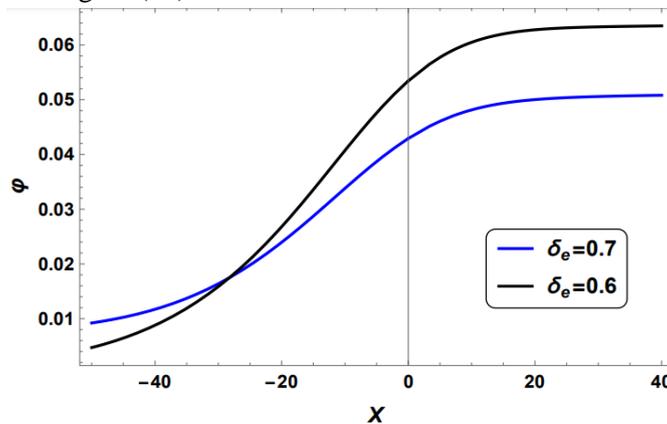
that an increase in electron temperature, rather than a decrease in ion temperature, can lead to an increase in the amplitude of a shock-like wave in the plasma as shown in the figure (1a).



**Fig. (1a)**

Figure 1: (a) The shock-like dust ion acoustic wave for different values of  $\sigma_i$ .

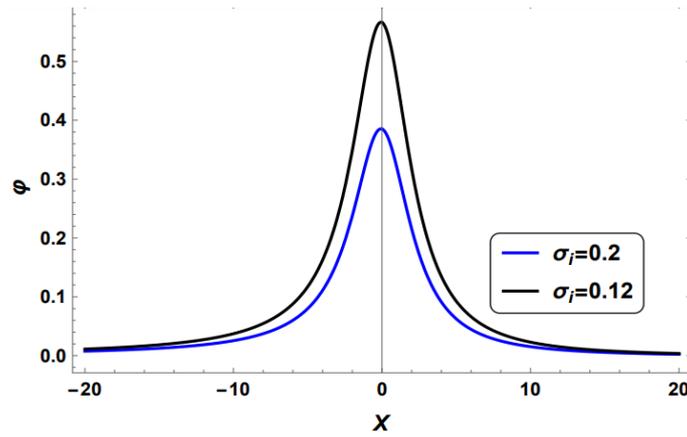
In plasma physics, electron numerical density refers to the concentration or abundance of electrons per unit volume. A shock-like wave is a fast, intense disturbance that propagates through a plasma physics. When the number density of electrons in a plasma physics environment decreases, there are fewer electrons in a given volume. The behavior of shock waves in plasma physics is affected by various factors, including the properties of the plasma and interactions between particles. In shock wave dynamics in plasma physics, the energy of a shock wave is related to the energy associated with the compression and heating of the plasma. That is, when the number density of electrons decreases, this means that there are fewer particles available to compress and interact, leading to a decrease in shock-like wave energy as shown in the figure (1b).



**Fig. (1b)**

Figure 1: (b) The shock-like dust ion acoustic wave for different values of  $\delta_e$ .

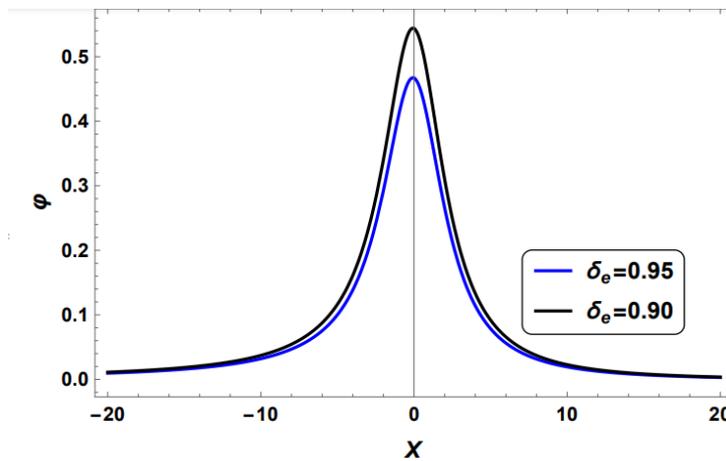
Figure (2a) shows us the behavior of the solitary wave, as the energy of the solitary wave increases when the temperature of the electrons increases, as shown above in the figure (1a).



**Fig. (2a)**

Figure 2: (a) The solitary dust ion acoustic wave for different values of  $\sigma_i$ .

Figure (2b) shows us the behavior of the solitary wave, as the energy of the solitary wave decreases when the density of the electrons decreases, as shown above in the figure (1a).



**Fig. (2b)**

Figure 2: (b) The solitary dust ion acoustic wave for different values of  $\delta_e$ .

In Figure (3a) we find that the behavior of the acoustic ion wave in the dusty plasma follows the same shock-like wave behavior as the single acoustic ion wave. This happens when the temperature of the positive ions increases, and also in figure (3b) when the number density of electrons increases.

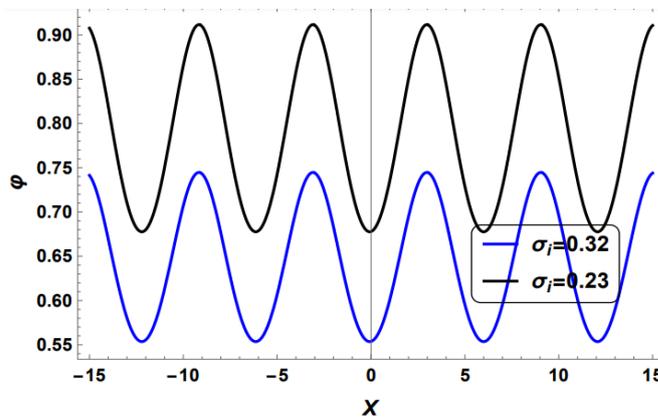


Figure 3: (a) The periodic dust ion acoustic wave for different values of  $\sigma_i$ .

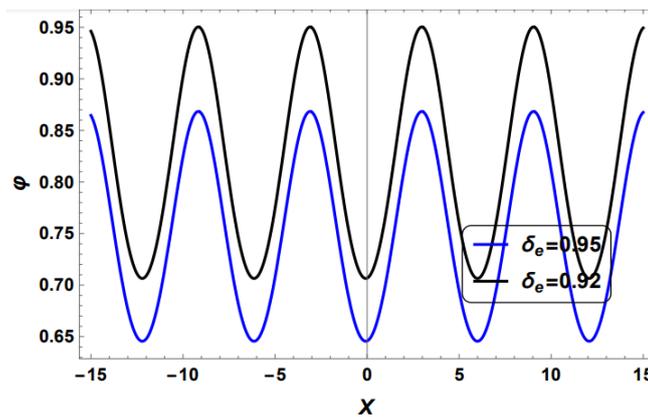


Figure 3: (b) The periodic dust ion acoustic wave for different values of  $\delta_e$ .

## 5. SUMMARY

Fluid nonlinear equations describe the plasma on the dark side of the Moon caused by interaction with the Earth's magnetosphere. We used perturbation theory to reduce the basic equations to the KdV equation. The last problem was solved analytically and studied numerically using the modified generalized - expansion methods. Different analytical solutions of the nonlinear evolution homogenous equation is obtained, which allow for the propagation of all both of solitary, shock-like and periodic pulses. Therefore, sound waves on the dark side of the Moon can support single pulses as well as shock-like and periodic pulses are depending on, the wave propagation plasmas parameter.

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