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Computing Exact Solution for Forward Linear Discrete Time Variant Systems

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ABSTRACT

In various engineering fields, such as circuits and electronics, solving discrete or digital systems is very important. By using linear algebra maple package, we will construct a simple procedure for computing the exact solution for linear discrete systems with time variable coefficients. We will give simple formulas for the solution of nonhomogeneous linear discrete systems for forward shift operators and forward difference operators. To illustrate our procedure, we will give some examples with different type of operators. By entering the parameters of the system in each example, we will receive the graph of exact solution.

Key Words: Discrete system, time-varying systems, difference equations, digital system, Maple programing.
Mathematics Subject Classification: 39A06, 93A60, 39A60

1. INTRODUCTION

Physicists and engineers have used the concept of system to study the relationship between matter and the force acting on it. A system is a mathematical expression designed to serve as a mathematical model of an interactive phenomenon. If the time domain is a continuous period, the system is known as a continuous time system and if the time domain is only for discrete moments of time t, where t varies across integer numbers, then the system is referred to as a discrete time system. We denote the continuous time function at time t by $f(t)$ and the discrete time function at time k should be denoted by $f(k)$.

Computer controls are generally carried out at discrete times depended on samples taken at discrete times. It is preferable to use discrete systems for several reasons, including that the computer language is the binary system. In addition, discrete-time models are usually much easier to identify than continuous-time models. [1-3]. Discrete-time models has been widely encountered in science and engineering fields such as compressed sensing, robot, optimal control and signal processing [4-15]. As well as finance and economics, many models have been defined to calculate the development of economic sizes across separate periods [16]. Some advantages and disadvantages of using discrete versus continuous systems can be found in [17].

Maple is a mathematical program that can solve complex mathematical problems by writing just a few lines of command. You can create your own procedures. The maple program contains important packages such as linear algebra package. In [18], we constructed a Maple procedure to get the exact solution of nonhomogeneous time-invariant continuous systems. Here, we will develop a Maple procedure for the solution of forward discrete system with variable coefficients. Throughout, we will use some commands like [19]:

> proc(.)...end proc:	# Procedure.
> with(LinearAlgebra)**:**	# Linear algebra package.
> seq($x(i), i = 0..m)$};	# Sequence $x(1), x(2), \ldots, x(m)$
> $A := matrix([\,[a,b,c],[d,e,f]])$;	# Defining 2 by 3 matrices.
> Transpose(A);	# Matrix transpose.
>MatrixInverse(A);	# Matrix inverse.
> Plot(v_1, v_2);	# Plot two vectors on one graph.

In addition, we shall use the following notations and definitions:

- R denotes the real line.
- $N_m = \{0,1,2,\ldots,m\}$
- $\prod_{\underline{i=\ell}}^{m} A(i) := \mathrm{A}(\ell).A(\ell+1).\ldots.\mathrm{A}(\mathrm{m}-1).\mathrm{A}(\mathrm{m}), \quad m \geq \ell$
- $\prod_{i=\ell}^{\underline{m}} A(i) := \mathrm{A}(\mathrm{m}).\mathrm{A}(\mathrm{m}-1)\ldots\mathrm{A}(\ell+1).A(\ell), \quad m \geq \ell$

Definition 1 *If* $k, k+m \in \mathbb{Z}$, *then for* $\mathrm{k} \in \mathrm{Z}$ *, we define the forward shift operator* $\boldsymbol{F}^m$, $m > 0$ *by*

$$\boldsymbol{F}^m(x(\mathrm{k})) = x(\mathrm{k}+\mathrm{m})$$

Definition 2 *If* $k, k+m \in \mathbb{Z}$, *then for* $k \in Z$ *, , we define the forward difference operators* $\boldsymbol{\Delta}^m$, $m > 0$ *by*

$$\boldsymbol{\Delta}(x(\mathrm{k}))) = (\boldsymbol{F}-\boldsymbol{I})x(\mathrm{k}) = x(\mathrm{k}+1) - x(\mathrm{k}),$$

$$\boldsymbol{\Delta}^2(x(\mathrm{k})) = \boldsymbol{\Delta}\left(\boldsymbol{\Delta}\ \ x(\mathrm{k})\right), \ldots. \quad \boldsymbol{\Delta}^m(x(\mathrm{k})) = \boldsymbol{\Delta}(\boldsymbol{\Delta}^{m-1}x(\mathrm{k})).$$

In this paper, we will construct a Maple procedure for computing the exact solution of nonhomogeneous linear discrete systems for forward shift operators and forward difference operators.

2. STATE OF DISCRETE SYSTEM

Let us consider the following forward linear system:

$$\begin{aligned}
&\boldsymbol{F}x_1(k)) = a_{11}(k)\,x_1(k) + a_{12}(k)\,x_2(k) + \cdots + a_{1n}(k)\,x_n(k) + u_1(k),\\
&\boldsymbol{F}(x_2(k)) = a_{21}(k)\,x_1(k) + a_{22}(k)\,x_2(k) + \cdots + a_{2n}(k)\,x_n(k) + u_2(k),\\
&\quad\vdots\\
&\boldsymbol{F}(x_n(k)) = a_{n1}(k)\,x_1(k) + a_{n2}(k)\,x_2(k) + \cdots + a_{nn}(k)\,x_n(k) + u_n(k),
\end{aligned}$$

with given initial conditions $x_1(k_0), \ldots, x_n(k_0)$

In compact form:

$$\textbf{(F-S)} \quad \begin{cases} F(x(k)) = A(k)\,x(k) + B(k)u(k), \\ \text{with given initial conditions} \\ x(k_0) = x_0, \qquad \mathrm{k}_0 \in N_m, \end{cases}$$

where $x(k) = [x_1(k), \ldots, x_n(k)]^T$, $u(k) = [u_1(k), \ldots, u_n(k)]^T$ and $A(\mathrm{k}) = [a_{ij}(k)]$ is $n \times n$ matrix with the following assumptions:

i. $u(k)$, $A(k)$ are defined for all $k \in N_m$ ii. $A(k)$ is invertible for all $k \in N_m$

Theorem 1 *For each* $k_0 \in N_m$ *and* $x(k_0) \in \mathfrak{R}^n$, *there exists a unique* $x(\mathrm{k}) \in \mathfrak{R}^n$ *solution of the system* **(F-S)** *under the assumptions (i-ii), this solution is given by*

$$(\mathbf{Solu-F-S})\quad x(k)=\begin{cases}\overline{\Phi}(A,k_0,k-1)\,.\,x(k_0)+\sum_{\tau=k_0}^{k-1}\overline{\Phi}(A,\tau+1,k-1).\,u(\tau) & if\quad k_0<k\leq \mathrm{m}\\ \underline{\Phi}(A^{-1},k,k_0-1).\,x(k_0)-\sum_{\tau=k}^{k_0-1}\underline{\Phi}(A^{-1},\tau,k).\,u(\tau) & if\ \ 0\ \leq k<k_0\end{cases}$$

where $\overline{\Phi}(A,\ell,m)=\prod_{\overline{i=\ell}}^{m}\mathrm{A(i)},\quad m\geq \ell\qquad \underline{\Phi}(A,\ell,m)=\prod_{\underline{\mathrm{i}=\ell}}^{m}\mathrm{A(i)},\quad m\geq \ell$

Proof. One may obtain the solution of **(F-S)** by a simple iteration:

1. If $k>k_0$, then

$$\begin{aligned}x(k_0+1)&=A(k_0)\,.\,x(k_0)+u(k_0),\\ x(k_0+2)&=A(k_0+1)\,.\,x(k_0+1)+u(k_0+1)\\ &=A(k_0+1)\,.\,A(k_0)\,.\,x(k_0)\\ &\quad +A(k_0+1)\,.\,u(k_0)+u(k_0+1)\end{aligned}$$

and

$$\begin{aligned}x(k_0+3)&=A(k_0+2)\,.\,x(k_0+2)+u(k_0+2)\\ &=A(k_0+2)\,.\,A(k_0+1)\,.\,A(k_0).\,x(k_0)\ +A(k_0+2)\,.\,A(k_0+1)\,.\,u(k_0)\\ &\quad +A(k_0+2)\,.\,u(k_0+1)+u(k_0+2)\end{aligned}$$

Inductively, it is easy to see that

$$x(k)=\left[\prod_{\overline{i=k_0}}^{k-1}\mathrm{A(i)}\right]x(k_0)+\sum_{\tau=k_0}^{k-1}\left[\prod_{\overline{i=\tau+1}}^{k-1}\mathrm{A(i)}\right]u(\tau)$$

2. If $k<k_0$, then

$$\begin{aligned}x(k_0)&=A(k_0-1)\,.\,x(k_0-1)+u(k_0-1)\\ \Rightarrow x(k_0-1)&=A^{-1}(k_0-1)\,.\,x(k_0)\\ &\quad -A^{-1}(k_0-1).\,u(k_0-1)\end{aligned}$$

$$\begin{aligned}x(k_0-2)&=A^{-1}(k_0-2)\,.\,A^{-1}(k_0-1).\,x(k_0)\\ &\quad -A^{-1}(k_0-2).\,A^{-1}(k_0-1)\,.\,u(k_0-1)\\ &\quad -A^{-1}(k_0-2).\,u(k_0-2)\end{aligned}$$

Inductively, it is easy to see that

$$x(k)=\left[\prod_{\overline{i=k}}^{k_0-1}\mathrm{A(i)}\right]^{-1}x(k_0)-\sum_{\tau=k}^{k_0-1}\left[\prod_{\overline{i=k}}^{\tau}\mathrm{A(i)}\right]^{-1}u(\tau)$$

Hence

$$x(k)=\begin{cases}\overline{\Phi}(A,k_0,k-1)\,.\,x(k_0)+\sum_{\tau=k_0}^{k-1}\overline{\Phi}(A,\tau+1,k-1).\,u(\tau) & if\quad k_0<k\leq m\\ \overline{\Phi}^{-1}(A,k,k_0-1).\,x(k_0)-\sum_{\tau=k}^{k_0-1}\overline{\Phi}^{-1}(A,\tau,k).\,u(\tau) & if\ \ 0\ \leq k<k_0\end{cases}$$

This complete the proof.

Remark 1. There are some special cases of **(F-S),** which are important in many applications:

1. If $A(k) = A(constant)$, then

$$x(k) = \begin{cases} \mathrm{A}^{k-k_0}\, x(k_0) + \sum_{\tau=k_0}^{k-1} \mathrm{A}^{k-\tau-1} \; u(\tau) \quad if \quad k_0 < k \leq \mathrm{m} \\ (\mathrm{A}^{-1})^{k_0-k} \quad x(k_0) + \sum_{\tau=k_0}^{k-1} (\mathrm{A}^{-1})^{\tau-k-1} \quad u(\tau) \quad if \;\; 0 \leq k < k_0 \end{cases}$$

2. If $k_0 = 0$, then

$$x(k) = \left[\prod_{i=0}^{k-1} \mathrm{A(i)}\right] x(0) + \sum_{\tau=0}^{k-1} \left[\prod_{i=\tau+1}^{k-1} \mathrm{A(i)}\right] u(\tau), \qquad 0 < k \leq m$$

3. If $k_0 = 0$, $A(k) = A$, then

$$x(k) = \mathrm{A}^{k}\, x(0) + \sum_{\tau=0}^{k-1} \mathrm{A}^{k-\tau-1} \;\; u(\tau), 0 < k \leq m$$

4. If $k_0 = 0$, $A(k) = A, u(k) = 0$ then

$$x(k) = \mathrm{A}^{k}\, y(0), 0 < k \leq m$$

5. If $k_0 = m$, then

$$x(k) = \left[\prod_{i=k}^{k_{\mathrm{f}}-1} \mathrm{A(i)}\right]^{-1} x(k_0) - \sum_{\tau=k}^{k_{\mathrm{f}}-1} \left[\prod_{i=k}^{\tau} \mathrm{A(i)}\right]^{-1} u(\tau), \qquad 0 < k \leq m$$

6. If $k_0 = m$, $A(k) = A$, then

$$x(k) = (\mathrm{A}^{-1})^{m-k}\, x(m) + \sum_{\tau=0}^{k-1} (\mathrm{A}^{-1})^{\tau-k-1} \;\; u(\tau), 0 < k \leq \mathrm{m}$$

7. If $k_0 = m$, $A(k) = A, u(k) = 0$ then

$$x(k) = (\mathrm{A}^{-1})^{m-k}\, x(\mathrm{m}), 0 < k \leq \mathrm{m}$$

Remark 2. *The natural response (homogeneous solution) depends on the initial condition $x(k_0)$ is given by*

$$x_h(k) \;= \begin{cases} \overline{\Phi}(A, k_0, k-1)\, x(k_0) \quad if \quad k\_0 < k \leq m \\ \underline{\Phi}(A^{-1}, k, k_0 - 1)\, x(k_0) \quad if \;\; 0 \leq k < k_0 \end{cases}$$

The forced response (particular solution) on the input signal $u(k)$ is given by

$$x_p(k) \;= \begin{cases} \sum_{\tau=k_0}^{k-1} \overline{\Phi}(A, \tau+1, k-1) u(\tau) \quad if \quad k\_0 < k \leq m \\ -\sum_{\tau=k}^{k_0-1} \underline{\Phi}(A^{-1}, \tau, k) u(\tau) \qquad if \;\; 0 \leq k < k_0 \end{cases}$$

And $\qquad x(k) = x_h(k) + x_p(k)$

Remark 3. *For each $x(k_0) \in \Re^n$, and $k_0 \in N_m$, there exists a unique* $x(\mathrm{k}) \in \Re^n$ *solution of*

(Δ -S) $\begin{cases} \mathbf{\Delta}(x(k)) = A(k)\, x(k) + u(k), \\ \text{with given initial conditions } \textit{given by} \\ x(k_0), \qquad \mathrm{k}_0 \in N_m \end{cases}$

$$(\mathbf{Solu} - \mathbf{\Delta} - \mathbf{S})\; x(k) = \begin{cases} \overline{\Phi}(A+I, k_0, k-1). x(k_0) + \sum_{\tau=k_0}^{k-1} \overline{\Phi}(A+I, \tau+1, k-1). u(\tau) \quad if \quad k\_0 < k \leq m \\ \underline{\Phi}((A+I)^{-1}, k, k_0-1). x(k_0) - \sum_{\tau=k}^{k_0-1} \underline{\Phi}((A+I)^{-1}, \tau, k). u(\tau) \quad if \;\; 0 \leq k < k_0 \end{cases}$$

3. HIGHER ORDER EQUATION AND METHODS

Here, we are interested in finding solutions for initial value problem of higher order forward equation, we transform scalar equations of order s into a s-dimensional system of first-order equations. Consider the equation

$$(\mathrm{F-HE})\quad \begin{cases} \sum_{i=0}^{s} a_i(k)F^i\, x(k) = \mathrm{u(k)}, \quad a_s(\mathrm{k}) = 1, \quad k \in N_{\mathrm{k_f}}, \quad n > m > 0. \\ x(k_0), Fx(k_0), F^2x(k_0), \dots, F^{s-1}x(k_0) \ \text{ are given}, \quad k_0 \in N_m \end{cases}$$

This relation may be written as a system of dimension m.

Let

$$\begin{aligned} z_0(k) &= x(k) \\ z_1(k) &= F\,x(k) = F\,z_0(k) \\ z_2(k) &= F^2\,x(k) = F\,z_1(k) \\ &\vdots \\ z_{s-1}(k) &= F^{s-1}\,x(k) = F\,z_{s-2}(k) \end{aligned}$$

Hence

$$\begin{aligned} F\,z_0(k) &= z_1(k), && z_0(k_0) \text{ is given} \\ F\,z_1(k) &= z_2(k), && z_1(k_0) \text{ is given} \\ &\vdots && \vdots \\ F\,z_{s-2}(k) &= z_{s-1}(k), && z_{s-2}(k_0) \text{ is given} \\ F\,z_{s-1}(k) &= -a_{s-1}(k)z_{s-1}(k) \dots - a_1(k)z_1(k) && \\ & && z_{s-1}(k_0) \text{ is given} \end{aligned}$$

Let $\mathbf{z}(\mathrm{k}) = (z_0(k), z_1(k), \dots, z_{s-1}(\mathrm{k}))$, in vector notation, we transcribe this system as

$$F\,\mathbf{z}(\mathrm{k}) \quad = \mathrm{A(k)}\,\mathbf{z}(\mathrm{k}) + \mathbf{u}(\mathrm{k}), \qquad \mathbf{z}(\mathrm{k}_0)\ is \text{ given} \qquad (4.1)$$

where

$$\mathrm{A(k)} = \begin{pmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ 0 & 0 & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 1 \\ -a_0(k) & -a_1(k) & -a_2(k) & \dots & -a_{s-1}(k) \end{pmatrix} \text{ and}$$

$$\mathbf{u} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 0 \\ u(k) \end{pmatrix}, \qquad \mathbf{z_0}(\mathrm{k}) = \begin{pmatrix} x(k_0) \\ Fx(k_0) \\ F^2x(k_0) \\ \vdots \\ \vdots \\ F^{s-1}x(k_0) \end{pmatrix},$$

4. THE MAPLE PACKAGE

Based on the previous sections, we can transform theorem 1 to algorithm for computing exact solution:

```
> Phi1:=proc(A::Matrix,n,m)
> local i,phi,psi;with(LinearAlgebra):
> for i from n by 1 to m do phi[n-1]:=1;phi[i]:=subs(k=i,A).phi[i-1];  end
do;
> end proc:
%.............................................
> Phi2:=proc(A::Matrix,n,m)
> local I,phi,psi;
> with(LinearAlgebra):
```

```
> for i from n by 1 to m do phi[n-1]:=1;phi[i]:=phi[i-
1].subs(k=i,MatrixInverse(A));  end do;
> end proc:
%.............................................>
Solu2:=proc(A::Matrix,k0,m,x0,u)
> local i,z,x,j,A1,n,u1,v,S,S1;
> with(LinearAlgebra):u1:=convert(u,list);n:=nops(u1);
S:={seq(i,i=0..m)};
> for i from 1 by 1 to n  do
>  for j from 1 by 1 to n  do for i from 0 by 1 to k0-1 do z[i]:=Phi2(A,i,
k0-1).x0- seq(Phi2(A,  i,j).subs(k=j,u),j=i..k0-1);
> end do ;
> for i from k0  to k0 do z[i]:=x0; end do;
> for i from k0+1 by 1 to m do z[i]:=Phi1(A, k0, i-1).x0+ seq(Phi1(A,  j+1,i-
1).subs(k=j,u),j=k0..i-1);
> end do;
> print(seq(x(i)=z[i],i=0..m));
> A1:=convert([seq(convert(z[i],list),i=0..m)],Matrix);
> A1:=Transpose(A1);v:=convert([seq(i,i=0..m)],Vector);
> for i from 1  to n do
>print(plot(v,convert(A1[i],Vector),style=patch,symbol=asterisk,symbolsize=10
0,color=blue));
> print(x[i] )
> end do
> end do
> end do;
> end proc:
```

5. EXAMPLES

To illustrate our procedure, we give some examples, which appear in engineering applications.

Example 1: Solve the following discrete system ($n = 1$, $k_0 = 2$, F operator):

$$F\big(x(k)\big) = \frac{k+2}{k+1}\,x(k) + k, \qquad k \in N_{10},$$
$$x(2) = 4$$

$$> \quad A := Matrix\left(\left[\left[\frac{k+2}{k+1}\right]\right]\right);\ u := Matrix([[k]]);\ x0 := Matrix([[4]]);\ Solu2(A, 2, 10, x0, u);$$

$$A := \left[\frac{k+2}{k+1}\right]\ u := [k]\ x0 := [4]$$

$$x(0) = [1], x(1) = [2], x(2) = [4], x(3) = \left[\frac{22}{3}\right], x(4) = \left[\frac{73}{6}\right],$$
$$x(5) = \left[\frac{93}{5}\right], x(6) = \left[\frac{267}{10}\right], x(7) = \left[\frac{1278}{35}\right], x(8) = \left[\frac{6731}{140}\right], x(9) = \left[\frac{7739}{126}\right], x(10) = \left[\frac{96469}{1260}\right]$$

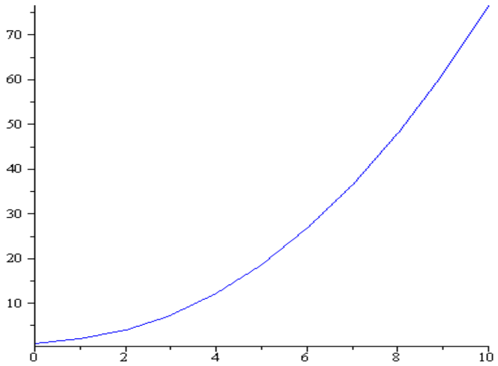


Figure 1. Solution of Example 1: $x(t)$

Example 2: Solve the following discrete system ($n = 1$, $k_0 = 0$, Δ operator):

$$\Delta\big(x(k)\big) = \frac{k+2}{k+1}\, x(k) + k, \qquad k \in N_{10},$$
$$x(0) = 1$$

$$> A := Matrix\left(\left[\left[\frac{k+2}{k+1} - 1\right]\right]\right); u := Matrix([[k]]); x0 := Matrix([[1]]); Solu2(A, 0, 10, x0, u);$$

$$A := \left[\frac{k+2}{k+1} - 1\right] \quad u := [k] \quad x0 := [1]$$

$$x(0) = [1], x(1) = [1], x(2) = \left[\frac{3}{2}\right], x(3) = \left[\frac{5}{2}\right], x(4) = \left[\frac{29}{8}\right], x(5) = \left[\frac{189}{40}\right], x(6) = \left[\frac{463}{80}\right], x(7) = \left[\frac{3823}{560}\right],$$
$$x(8) = \left[\frac{35183}{4480}\right], x(9) = \left[\frac{357743}{40320}\right], x(10) = \left[\frac{3986543}{403200}\right]$$

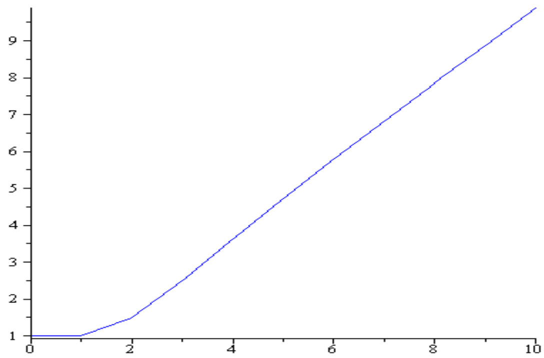
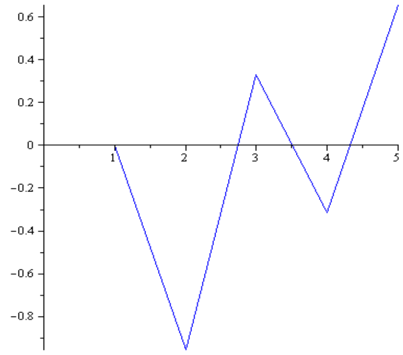


Figure 1. Solution of Example 2: $x(t)$

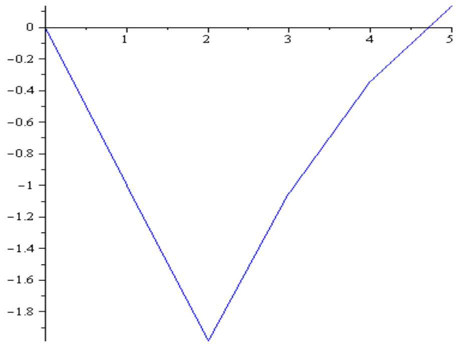
Example 3: Solve the following discrete system ($n = 3$, $k_0 = 0$, F operator):

$$A := \begin{bmatrix} 0.3 - 0.9\sin(10k) & 0.1 & 0.7\cos(10k) \\ 0.6\sin(10k) & 0.3 - 0.8\cos(10k) & 0.01 \\ 0.5 & 0.15 & 0.6 - 0.9\sin(10k) \end{bmatrix}$$

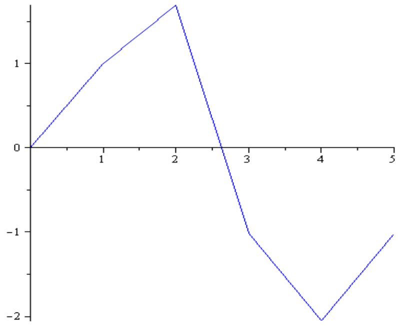
$$u := \begin{bmatrix} 0.5\sin(12k) \\ 0.5\sin(12k) - \cos(7k) \\ \cos(7k) \end{bmatrix} \quad x0 := \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$



$x_1(t)$



$x_2(t)$



$x_3(t)$

Figure 3. Solutions of Example 3: $x_1(t)$, $x_2(t)$ and $x_3(t)$

Example 4: Solve the following discrete system ($n = 1$, $k_0 = 0$, F^3 operator):

$$x(k+3) + x(k+2) - k\,x(k+1) - x(k) + k = 0, \qquad k \in N_7,$$
$$x(0) = 1, \quad x(1) = 2, \quad x(2) = -1$$

```
> A := Matrix([[0, 1, 0], [0, 0, 1], [1, k,−1]]); u := Matrix([[0], [0], [
    −k]]); x0 := Matrix([[1], [2], [−1]]); Solu2(A, 0, 7, x0, u);
```

$$A := \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & k & -1 \end{bmatrix} u := \begin{bmatrix} 0 \\ 0 \\ -k \end{bmatrix} x0 := \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$$

$$x(0) = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}, x(1) = \begin{bmatrix} 2 \\ -1 \\ 2 \end{bmatrix}, x(2) = \begin{bmatrix} -1 \\ 2 \\ -2 \end{bmatrix}, x(3) = \begin{bmatrix} 2 \\ -2 \\ 3 \end{bmatrix}, x(4)$$
$$= \begin{bmatrix} -2 \\ 3 \\ -10 \end{bmatrix}, x(5) = \begin{bmatrix} 3 \\ -10 \\ 16 \end{bmatrix}, x(6) = \begin{bmatrix} -10 \\ 16 \\ -68 \end{bmatrix}, x(7) = \begin{bmatrix} 16 \\ -68 \\ 148 \end{bmatrix}$$

So $x(0) = 1$, $x(1) = 2$, $x(2) = -1$, $x(3) = 2$, $x(4) = -2$, $x(5) = 3$, $x(6) = -10$, $x(7) = 16$.

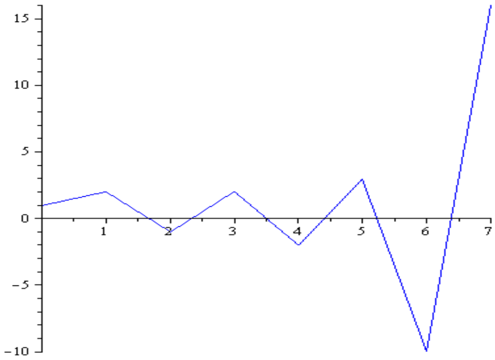


Figure 4. Solution of Example 4: $x(t)$

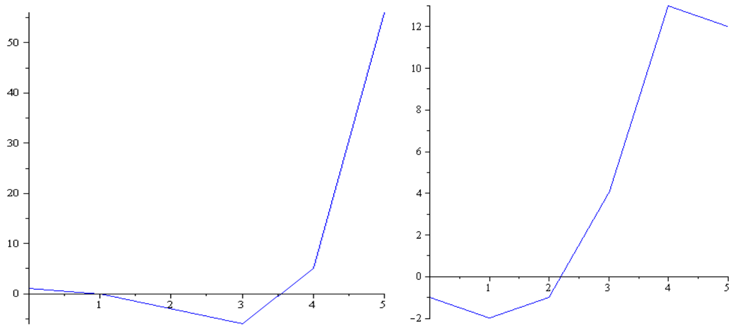
Example 5: Solve the following discrete system ($n = 2$, $k_0 = 0$, F operator):

$$x_1(k+1) = x_1(k) + k\,x_2(k) - 1, \qquad k \in N_5,$$
$$x_2(k+1) = x_2(k) - x_1(k) + k$$
$$x_1(0) = 1, \quad x_2(0) = -1$$

```
> A := Matrix([[1, k], [-1, 1]]); u := Matrix([[-1], [k]]); x0
      := Matrix([[1], [-1]]); Solu2(A, 0, 5, x0, u);
```

$$A := \begin{bmatrix} 1 & k \\ -1 & 1 \end{bmatrix} u := \begin{bmatrix} -1 \\ k \end{bmatrix} x0 := \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$x(0) = \begin{bmatrix} 1 \\ -1 \end{bmatrix}, x(1) = \begin{bmatrix} 0 \\ -2 \end{bmatrix}, x(2) = \begin{bmatrix} -3 \\ -1 \end{bmatrix}, x(3) = \begin{bmatrix} -6 \\ 4 \end{bmatrix}, x(4) = \begin{bmatrix} 5 \\ 13 \end{bmatrix}, x(5) = \begin{bmatrix} 56 \\ 12 \end{bmatrix}$$



$x_1(t)$ $x_2(t)$

Figure 5. Solutions of Example 5: $x_1(t)$ **,** $x_2(t)$

6. CONCLUSION

In the paper, a Maple procedure was constructed for computing the exact solutions of nonhomogeneous forward discrete linear systems with variable coefficients. The advantage of this procedure is that the solution is obtained directly by entering the parameters only.

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