Alfarama Journal of Basic & Applied Sciences

Faculty of Science Port Said University

April 2024, Volume 5, Issue II (Special Issue)

ISSN 2682-275X

https://ajbas.journals.ekb.eg
ajbas@sci.psu.edu.eg

http://sci.psu.edu.eg/en/

DOI:https://doi.org/10.21608/AJBAS.2024.241492.1190

Submitted: 08/10/2023
Accepted: 15/02/2024

Pages: 262 - 280

Prediction Using Markov Model and Hidden Markov Model for the States of Banknotes

Mohammed El Genidy[1], Mosaad Hendam[1,*], Khaled Mahfouz[1]

[1]Dept. of Math and Computer Sci., Faculty of Science, Port Said Univ.,

***Corresponding author:** mosaadhendam@gmail.com

ABSTRACT

This study aims to solve the problem of identifying the values of banknotes, and whether they are fake or not. This problem is considered one of the topics that many researchers are interested in the field of detecting banknotes values and determining whether they are fake. For this reason, Markov models were applied to identify the probabilities of banknote values and whether they are fake or not. In addition, the hidden Markov model was used to identify the probabilities of banknotes states when they were deposited a limited number of times. Important numerical results were obtained in applying the Markov model and the hidden Markov model where it can be implemented in various other applications to be used in prediction. This study discovered the probabilities of banknote values and whether they are fake or not when depositing using the Markov models. In addition, the probabilities of tracing the trajectories of banknote states on deposit were obtained by the hidden Markov model. Researchers can apply this study to other data sets to predict the future data.

Key Words:

Markov Model, Hidden Markov Model, Bayes theorem, Transition matrix.

1. INTRODUCTION

The mechanism of reciprocal benefit exchange pertinent to daily human activities has undergone a significant evolution, transitioning from primitive barter systems to the advent of currency as a medium for commercial transactions. This currency has experienced various developmental phases, culminating in its present form. Presently, banknotes have attained considerable significance, primarily due to the widespread confidence in their legitimacy and security, thereby facilitating their acceptance in transactions without apprehension regarding their authenticity or safety [1].

The proliferation of counterfeiting activities has raised concerns about the gradual erosion of trust in paper currencies, potentially leading to significant economic repercussions. Consequently, there has been an escalated legislative focus on criminalizing the act of forgery [2]. Particularly noteworthy is the substantial advancement in the methodologies employed for counterfeiting banknotes. These advancements have enabled the production of counterfeit paper currency with remarkable precision. This issue is exacerbated in numerous nations where banknotes are printed using low-quality presses and non-standard ink types. As a result, their authentic currency often exhibits characteristics typically associated with counterfeit banknotes, further complicating the issue of currency validation and trust [3].

El-Genidy et al. employed alternative prediction methodologies by utilizing estimation regression model algorithms and subsequently converting the data set into a Markov model. This approach was applied in the context of a study focusing on water quality in the northern part of Lake Manzala over the period from January to March 2016. To assess and forecast the condition of the water in Lake Manzala, a sophisticated model encompassing both linear and non-linear regression analyses was developed, specifically tailored to the water elements present in the lake [4].

Markov models have been implemented to ascertain the likelihood of banknote denominations and to determine their authenticity, discerning between genuine and counterfeit notes. This methodology builds upon prior research in the realm of facial recognition, where the Hidden Markov Model was effectively utilized to identify seven distinct facial expressions: anger, disgust, fear, joy, sadness, surprise, and neutrality. The foundation of this approach lies in the utilization of engineering features pertinent to effective facial expression recognition. Such an approach has significant potential to enhance human-computer interaction by providing a more nuanced and accurate interpretation of facial expressions [5].

Conversely, a pragmatic model employing the Hidden Markov Model has been developed for the detection of suspicious financial transactions. This model is particularly relevant given the clandestine nature of such transactions, which are often deliberately obscured from financial institutions. The Hidden Markov Model is designed to ascertain the likelihood of a transaction being suspicious. The efficacy of this model is evaluated using datasets comprising both actual and dubious transactions [6]. Additionally, there has been notable research in the development of a currency detection system specifically designed for the visually impaired in India. This system leverages a vision-based approach to aid in the recognition of banknotes, thereby significantly enhancing the autonomy of blind individuals in financial transactions. The implementation of this recognition system is achieved through an array of sophisticated machine learning and deep learning techniques [7].

jang et al. have made a significant contribution by developing an automated system for the identification of banknote serial numbers. This system is based on a visual letter recognition framework that utilizes deep learning techniques. Such an approach enables efficient and accurate recognition of serial numbers on banknotes, streamlining processes that require such identification [8]. Furthermore, Ilham et al. have innovatively constructed a speech and visual recognition system tailored for the Amazigh language. This system uniquely integrates auditory and visual data, enhancing its effectiveness in language recognition. Key characteristics are extracted specifically from the mouth region and are then analytically processed using Hidden Markov Models. This dual-modality approach significantly improves the accuracy and reliability of the system in recognizing and interpreting the Amazigh language [9].

A notable advancement in the field of biometric systems is the proposal of a face recognition system, wherein Singular Value Decomposition (SVD) is employed for feature extraction, and the Hidden Markov Model (HMM) is utilized for the classification process. This approach leverages the strength of

SVD in effectively reducing the dimensionality of facial data while retaining essential features, coupled with the HMM's capability for dynamic pattern recognition, thereby enhancing the overall efficiency and accuracy of the face recognition process [10]. In a similar vein, a novel coin recognition method has been introduced. This method is predicated on modeling the texture properties of coins using a Hidden Markov Model. What sets this approach apart is its ability to differentiate between coins from various countries, primarily through the analysis of their unique textural characteristics. This technique not only enhances the accuracy of coin recognition but also broadens its applicability across different national currencies [11].

A significant advancement in the realm of speech recognition has been achieved through the implementation of a system based on the Hidden Markov Model, which incorporates energy transfer techniques. This approach has notably enhanced the accuracy of speech recognition results. By effectively capturing and processing the nuances of speech patterns, the system represents a considerable improvement over traditional models, leading to more reliable and precise speech recognition capabilities [12].

Human-computer interaction, previous research has focused on the application of the Hidden Markov Model in gesture recognition, aimed to diminish the communication barrier between computers and humans by interpreting and understanding human gestures more effectively. The model was meticulously programmed and underwent extensive improvements. It integrated the outputs of a network to derive the final prediction class, benefitting from comprehensive training and refinements. This approach not only improved the accuracy of gesture recognition but also enhanced the interaction between users and computer systems, making it more intuitive [13,14].

Concurrently, in the field of linguistic technology, a noteworthy achievement has been made by researchers in accurately identifying various Arabic dialects. A specialized system has been proposed and developed for the recognition of a specific set of regional and contemporary Arabic dialects. This system is grounded in speech recognition technology, utilizing Hidden Markov Model algorithms. It operates by receiving verbal words as input and generating the corresponding spoken language as output. During the crucial training phase of this system, utterances from one or more Arabic dialects were meticulously analyzed. This analysis was aimed at capturing key characteristics of the vocal signals, particularly focusing on aspects related to time and frequency. By doing so, the system was able to distinguish between the subtle nuances that differentiate various Arabic dialects, thereby enhancing its accuracy and effectiveness in dialect recognition [15].

In the present study, the Markov model has been adeptly utilized to predict the likelihood of banknotes being counterfeit or genuine, particularly when handling two distinct types of banknotes. This application of the Markov model plays a crucial role in enhancing the accuracy and efficiency of detecting fraudulent currency transactions. Furthermore, Hidden Markov Chains have been employed as a strategic tool in this study to ascertain the most probable path or the highest likelihood of encountering either counterfeit or damaged banknotes. This method was applied across a range of currencies within specified categories. The use of Hidden Markov Chains allows for a more nuanced and sophisticated analysis of currency authenticity, effectively identifying patterns and probabilities that may not be immediately apparent. This approach is particularly valuable in the context of handling multiple currencies, where variations in security features and conditions of banknotes can be complex and diverse.

2. MATERIALS AND METHODS

Markov chains represent a fundamental mathematical concept, pivotal in modeling the progression of sequential states over time. The foundational principle of these chains is the assumption that the probability of transitioning into a future state is solely dependent on the present state and is independent of the historical sequence of states. This characteristic renders Markov chains as a versatile tool in diverse disciplines, including probability theory, computer science, economics, and psychology [16]. The structure of a Markov chain comprises a finite number of possible states, along with a defined set of transitions between these states. These transitions are quantitatively described in a transition matrix, which assigns probability values to each possible state change. The transition matrix serves as a crucial tool for calculating the likelihood of transitioning into a specific state in the future, based on the current state of the system [17].

The applicability of Markov chains extends to various practical scenarios, such as weather forecasting, financial market analysis, predicting patterns in transportation networks, and analyzing job applicant resumes. For instance, in meteorology, Markov chains can be employed to forecast the weather, predicting subsequent weather conditions based on the current state and the probabilities of different weather transitions [18]. Overall, Markov chains offer a robust framework for analyzing and forecasting the dynamics of state changes over time across numerous fields. Their ability to provide insights into the likelihood of future states, grounded in the present state and its transition probabilities, makes them an analytical tool for a wide range of predictive modeling applications [19].

Consider X_t is a stochastic process, t =1, 2, 3, … . Where X_t is the state at time t, $X_t = 5$, it is mean the process in state 5 at time t. Where X_t is a random variable with discrete-time stochastic. Markov property means that X_{t+1} depends upon X_t, but it does not depend upon $X_{t-1}, \ldots, X_1, X_0$ as follows:

$$P(X_{t+1} = j \mid X_t = i_t, X_{t-1} = i_{t-1}, \ldots, X_0 = i_0) = P(X_{t+1} = j \mid X_t = i_t) \quad (1)$$

for all t = 1, 2, 3, . . . and for all states $i_t, i_{t-1}, i_{t-2,\ldots\ldots\ldots\ldots} i_{0.}$

Suppose that X_t be a sequence of discrete random variables where t =1, 2, 3, … . Consequently, X_t, is called a Markov chain of order n, if it satisfies the Markov property (known as memoryless property), then:

$$P(X_{t+1} = j \mid X_t = i_t, \ldots\ldots, X_n = i_n) = P(X_{t+1} = j \mid X_t = i_t, X_{t-1} = i_{t-1}, \ldots, X_0 = i_0) \quad (2)$$

The transition matrix P is sized n×n, with the element Pij indicating the likelihood of transitioning from state i to state j.

$$P = (p_{ij}), p_{ij} \geq 0 \text{ for all } i, j \in S \quad (3)$$

Where

$$\sum_{j=1}^{N} P_{ij} = \sum_{j=1}^{N} P(X_{t+1} = j \mid X_t = i) = 1 \quad (4)$$

$$P_{ij} = \begin{bmatrix} p_{11} & p_{12} & \cdots & p_{1j} \\ p_{21} & p_{22} & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots \\ p_{i1} & p_{i2} & \cdots & p_{ij} \end{bmatrix} \quad (5)$$

While the initial state matrix given by:

$$\pi = (\pi_i, i \in S), \text{ with } \pi_i = P(X_0 = i) = \begin{bmatrix} \pi_1 \\ \vdots \\ \pi_N \end{bmatrix} \quad (6)$$

The m-step transition probabilities, Let $\{X_0, X_1, X_2, \ldots\}$ be a Markov chain with state space $S = \{1, 2, \ldots, N\}$.
the probabilities of moving from state i to state j in m steps is as follows:

$$p_{ij}^{m} = P(X_{t+m} = j | X_t = i) \quad (7)$$

Suppose that $\{X_0, X_1, X_2, \ldots\}$ be a Markov chain with N × N transition matrix P.
Then the m-step transition probabilities are given by the matrix P^m:

$$P(X_m = j | X_0 = i) = (p^m)_{ij} \quad (8)$$

The Hidden Markov Model (HMM) enhances traditional Markov chains by incorporating hidden states that produce observable outputs, proving instrumental in fields such as speech recognition and natural language processing. These models adeptly navigate scenarios where the underlying states, like phonemes in speech or syntactic states in text, cannot be directly observed. HMMs consist of both hidden and observable states and model transitions between hidden states with probabilities similar to those in standard Markov chains. This allows for the analysis of sequential data that has an underlying hidden structure. A key goal within HMMs is to identify the most likely sequence of hidden states given the observed outputs. Techniques like the Viterbi algorithm play a crucial role here, using known transition and emission probabilities to deduce probable sequences. This capacity to infer hidden structures from observable data makes HMMs particularly effective in scenarios where directly observing the underlying processes is impractical, thus providing significant insights into the analysis of sequential data [20,21].

Hidden Markov Models (HMMs) are characterized by several key equations. The transition probabilities a_{ij} determine the likelihood of transitioning from one hidden state S_i to another S_j. These probabilities are organized into a transition matrix A. Emission probabilities $bj(k)$ represent the probability of observing event v_k given the hidden state Sj, forming the emission matrix B. Additionally, the initial state probabilities π_i describe the likelihood of starting in a particular hidden state Si at the initial time step, forming the initial state probability matrix π.

Hidden States and Observations:

- Hidden States: $S=\{S_1,S_2,...,S_N\}$ - set of hidden states.
- Observable Events: $V=\{v_1,v_2,...,v_M\}$ - set of observable events

Transition Probabilities:

- $a_{ij}=P(q_{t+1}=S_j|q_t=S_i)$ - probability of transitioning from state Si to state Sj.
  Transition Matrix: $A=[a_{ij}]_{N\times N}$

Emission Probabilities:

- $b_j(k)=P(O_t=v_k|q_t=S_j)$ - probability of observing event v_k given state S_j.
- Emission Matrix: $B=[bj(k)]_{N\times M}$

Initial State Probabilities:

- $\pi_i = P(q_1 = S_i)$ - probability of starting in state S_i at time t=1.
- Initial State Probability Vector: $\pi = [\pi i]_{1\times N}$

State Transition Equations:

- $P(q_{t+1}=S_j)=\sum_{i=1}^{N} P(q_{t+1}=S_j|q_t=S_i)\cdot P(q_t=S_i)$.

These equations define the fundamental components and dynamics of HMMs, enabling researchers to model and analyze sequences of observable events generated by hidden states

Definition: Suppose that the set of M possible observed events as $V=\{v_1, v_2,..,v_M\}$ that hidden states S produce. Therefore, the hidden sequence of $Q = q_1, q_2, .,q_T$ emits a sequence of T observations $O = O_1, O_2, . . . , O_T$ [22]. Moreover, the emission probability matrix of the hidden states at time t, $q_t = S_i$, produced an observed event $O_t=v_t$where the matrix of observation given by:

$$\mathrm{B} = \{b_i(v_k)\} = \begin{pmatrix} b_1(v_1) & b_1(v_2) & \dots & b_1(v_M) \\ b_2(v_1) & b_2(v_2) & \dots & b_1(v_M) \\ \vdots & \vdots & \ddots & \vdots \\ b_i(v_k) & b_i(v_k) & \dots & b_i(v_k) \end{pmatrix}, b_i(v_k) = P(O_t = v_k|q_t = S_i) \tag{9}$$

Where

$$\sum_{\mathrm{K}=1}^{\mathrm{M}} \mathrm{b_i(v_k)} = 1 \tag{10}$$

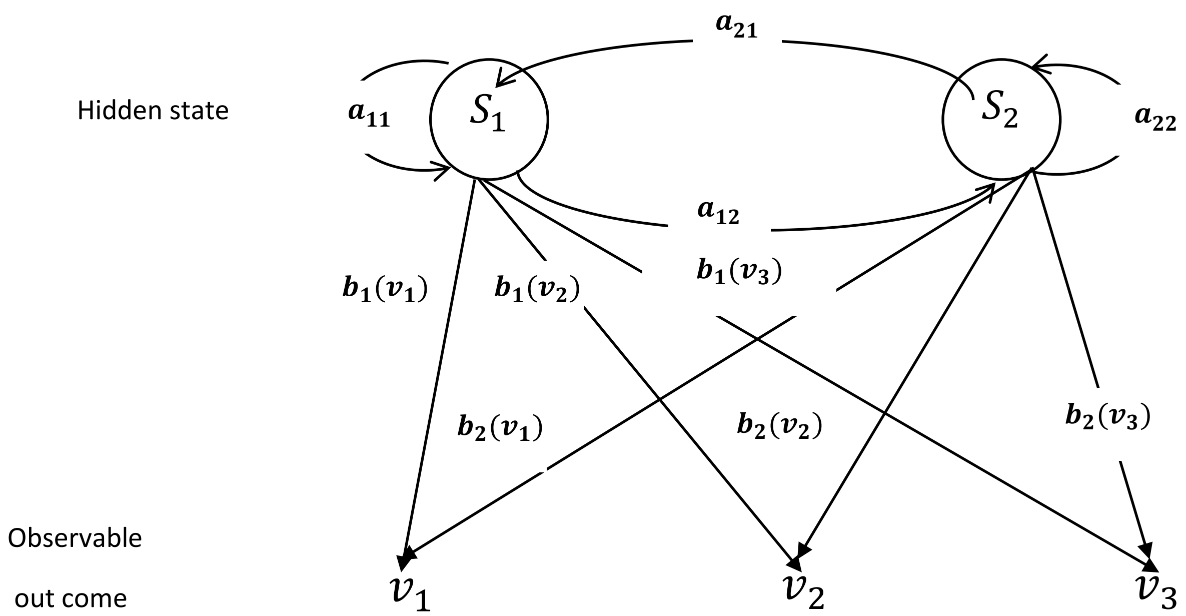


Fig. (1): Hidden Markov Model with three possible emission and two hidden states

Fig. (1) show the model where S_1 and S_2 are hidden states, $v1, v2, v3$ are observable states, aij is transition probability, $bi(vk)$ is emission probability. The HMM in Fig (1) generates a sequence of hidden states

$Q = q_1, q_2, \dots, q_T$ and for each a corresponding observation.

Example for hidden Markov model.

Suppose we have a weather forecasting system that predicts whether the weather is rainy or sunny. However, we don't have direct observations of the weather; instead, we observe people's activities, which are influenced by the weather. We want to model both the underlying weather states (hidden states) and the observed activities (observations).

Hidden States (Weather Conditions):

- Sunny (S)
- Rainy ®

Observations (People's Activities):

- Umbrella (U): People carry umbrellas when it's raining.
- No Umbrella (N): People don't carry umbrellas when it's sunny.

Transition Probabilities:

- a_{ij} represents the probability of transitioning from state i to state j
- Transition matrix A:

$$A = \begin{bmatrix} \boldsymbol{P(S \to S)} & \boldsymbol{P(R \to S)} \\ \boldsymbol{P(S \to R)} & \boldsymbol{P(R \to R)} \end{bmatrix}$$

For example:

- $P(S\to S)$=0.8
- $P(R\to S)$=0.2
- $P(S\to R)$=0.4
- $P(R\to R)$=0.6

$$\text{A}= \begin{bmatrix} 0.8 & 0.2 \\ 0.4 & 0.6 \end{bmatrix}$$

Emission Probabilities:

- $bj(k)$ represents the probability of observing symbol k from state j
- Emission matrix B:

$$B = \begin{bmatrix} P(U \mid S) & P(N \mid S) \\ P(U \mid R) & P(N \mid R) \end{bmatrix}$$

For example:

- $P(U|S)$=0.1
- $P(N|S)$=0.9
- $P(U|R)$=0.8
- $P(N|R)$=0.2

Initial State Probabilities:

- π represents the initial state probabilities matrix:

$$\pi=[P(S_0) \quad P(R_0)]$$

For example:

- $P(S_0)$ = 0.5 Initial probability of being sunny
- $P(R_0)$ = 0.5 Initial probability of being rainy

We want to find the probability of the weather sequence S,S,R,S given the observation sequence U,U,N,U. Now, let's apply the forward algorithm:

Initialization:

Initialize the forward variable α for each state at time t=0 using the initial probabilities:

- $\alpha_0(S)=\pi(S)\times b_S(U)=P(S_0)\times P(U|S)$
- $\alpha_0(R)=\pi(R)\times b_R(U)=P(R_0)\times P(U|R)$
- $P(S_0)\times P(U|S)=0.5\times 0.1=0.05$
- $P(R_0)\times P(U|R)=0.5\times 0.8=0.4$

Induction:

$\alpha_1(S)=[(\alpha_0(S)\times a_{SS})+(\alpha_0(R)\times a_{RS})]\times b_S(U)$

$\alpha_1(R)=[(\alpha_0(S)\times a_{SR})+(\alpha_0(R)\times a_{RR})]\times b_R(U)$

$\alpha_2(S)=[(\alpha_1(S)\times a_{SS})+(\alpha 1(R)\times aRS)]\times b_S(N)$

$\alpha_2(R)=[(\alpha 1(S)\times aSR)+(\alpha 1(R)\times aRR)]\times bR(N)$

$\alpha_3(S)=[(\alpha 2(S)\times aSS)+(\alpha 2(R)\times aRS)]\times bS(U)$

$\alpha_3(R)=[(\alpha 2(S)\times aSR)+(\alpha 2(R)\times aRR)]\times bR(U)$

$\alpha_1(S)=[(0.05\times0.8)+(0.4\times0.2)]\times P(U|S)=(0.04+0.08)\times0.1=0.012$

$\alpha_1(R)=[(0.05\times0.4)+(0.4\times0.6)]\times P(U|R)=(0.02+0.24)\times0.8=0.192$

$\alpha_2(S)=[(0.012\times0.8)+(0.192\times0.2)]\times P(N|S)=(0.0096+0.0384)\times0.9=0.04224$

$\alpha_2(R)=[(0.012\times0.4)+(0.192\times0.6)]\times P(N|R)=(0.0048+0.1152)\times0.2=0.024$

$\alpha_3(S)=[(0.04224\times0.8)+(0.024\times0.2)]\times P(U|S)=(0.033792+0.0048)\times0.1=0.0035592$

$\alpha_3(R)=[(0.04224\times0.4)+(0.024\times0.6)]\times P(U|R)=(0.016896+0.0144)\times0.8=0.0250368$

Termination:

$P(U,U,N,U)=\alpha_3(S)+\alpha_3(R)$

$P(U,U,N,U)=\alpha_3(S)+\alpha_3(R)=0.0035592+0.0250368=0.028596$

So, the probability of the observation sequence U,U,N,U given the Hidden Markov Model is approximately 0.0286.

3. THE MAIN RESULT

PREDICTION USING MARKOV MODEL AND HIDDEN MARKOV MODEL.

Detection of the fake (F) and not fake (T) of the banknotes by Markov model

To calculate the probability of a banknote being fake (F) or not fake (T) using a Markov model, we first need to establish the transition probabilities between these two states. In a Markov model, the future state depends only on the current state and not on the sequence of events that preceded it.

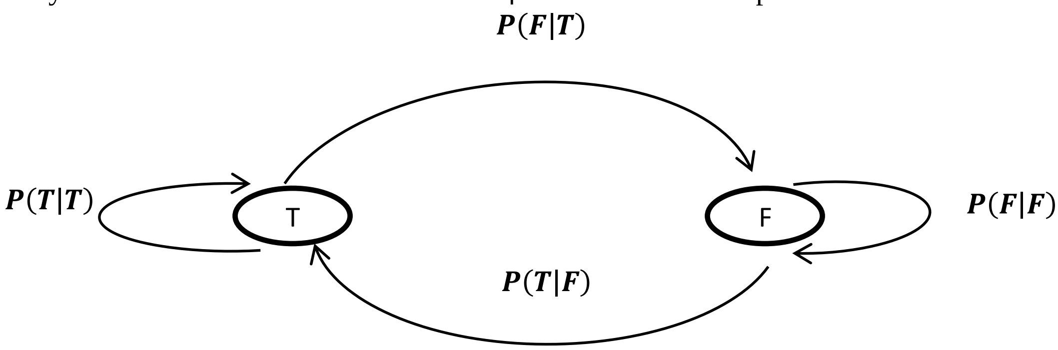


Fig (2): Markov model of the types of the banknotes fake (F) and not fake (T)

To define the transition matrix based on the given states "fake" (F) and "not fake" (T) for banknotes, as shown in Fig (2), we need to specify the probabilities of transitioning from each state to both states. The transition matrix is a square matrix where each element represents the probability of moving from one state to the other. It is typically structured as follows:

$$K=\begin{bmatrix} P(T|T) & P(T|F) \\ P(F|T) & P(F|F) \end{bmatrix} \tag{11}$$

The initial states matrix in a Markov model represents the starting probabilities of being in each state. For the banknote example with states "fake" (F) and "not fake" (T), the initial states matrix (or vector) can be represented as follows:

$$I=\begin{bmatrix} P(T) \\ P(F) \end{bmatrix} \tag{12}$$

Detection the type of the banknotes A and B by using Markov model.

To determine the type of banknotes, A or B, after they have been deposited multiple times, a Markov model can be utilized. In this scenario, the states of the Markov model would represent the types of banknotes. Let's define the states as follows:

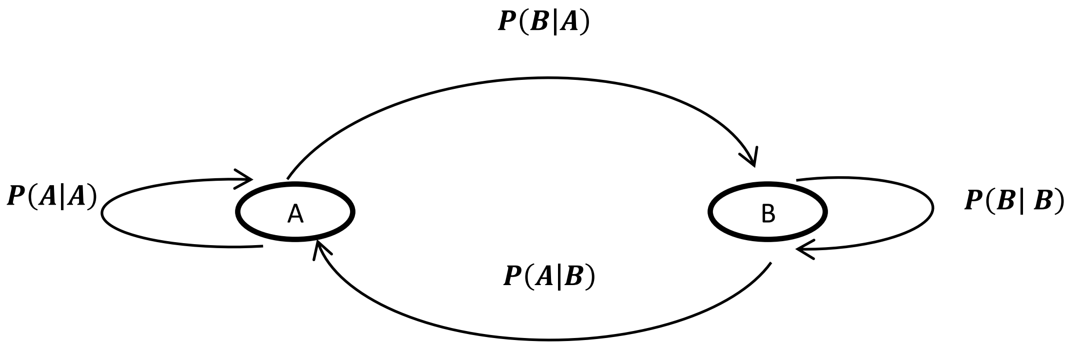


Fig (3): Markov model of the values of banknotes A and B

Fig (3) illustrates the transition states for banknotes of values A and B, then you can define the transition matrix based on this information. The transition matrix should reflect the probabilities of transitioning from each banknote type to another upon each deposit. It will be structured as a 2x2 matrix, where each element represents the probability of moving from one state (banknote type) to the other.

$$a_{ij}=\begin{bmatrix} P(A|A) & P(A|B) \\ P(B|A) & P(B|B) \end{bmatrix} \tag{13}$$

And the initial states matrix is as follows:

$$\pi = \begin{bmatrix} P(A) \\ P(B) \end{bmatrix} \tag{14}$$

From Bayes theorem

$$\mathrm{P(X|Y)} = \frac{\mathrm{P(Y|X).P(X)}}{\mathrm{P(Y)}} \tag{15}$$

Where P(X|Y) is the probability of X given Y, P(Y|X) is the probability of Y given X, P(X) is the probability of X, and P(Y) is the probability of Y.

Bayes' theorem is a cornerstone of statistical inference, widely applied across diverse fields such as machine learning, artificial intelligence, medical diagnosis, and spam filtering. This theorem forms the basis for Bayesian inference, a method that elegantly blends prior knowledge with newly observed data, allowing for dynamic updating of beliefs and predictions. In machine learning, it enables systems to adapt and learn from new information. In medicine, it assists in diagnosing diseases by integrating known disease probabilities with patient symptoms and test results. In digital communication, particularly for spam detection, it helps in filtering unwanted emails by analyzing the frequency and patterns of specific words [23].

Applying Bayes theory to obtain P(A|B) and P(T|F)as follow:

$$\mathrm{P(A|B)} = \frac{\mathrm{P(B|A).P(A)}}{\mathrm{P(B)}} \tag{16}$$

$$\mathrm{P(T|F)} = \frac{\mathrm{P(F|T).P(T)}}{\mathrm{P(F)}} \tag{17}$$

Hidden Markov Model for two types of banknotes A, B and Observable fake (F) and not fake (T)

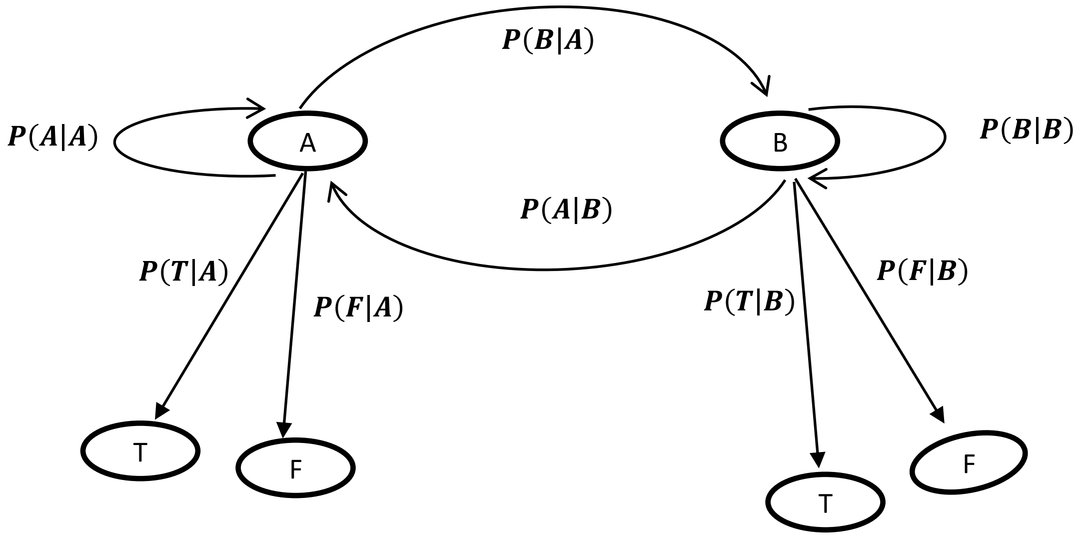


Fig (4): Hidden Markov Model for two types of banknotes A and B

Bayes' theorem can be applied to predict the type of a banknote (A or B) given its authenticity status (fake or not fake), as depicted in Fig (4) and based on equations (16) and (17). Bayes' theorem is particularly useful in this context as it allows for updating the probability of a hypothesis (in this case, the type of banknote) based on new evidence (whether the banknote is fake or not).

$$Pr(\{path\ 1\}) = P(A).P(T|A).P(B|A).P(T|B) \quad (18)$$

$$Pr(\{path\ 2\}) = P(A).P(F|A).P(B|A).P(F|B) \quad (19)$$

$$Pr(\{path\ 3\}) = P(A).P(T|A).P(B|A).P(F|B) \quad (20)$$

$$Pr(\{path\ 4\}) = P(A).P(F|A).P(B|A).P(T|B) \quad (21)$$

$$Pr(\{path\ 5\}) = P(B).P(T|B).P(A|B).P(T|A) \quad (22)$$

$$Pr(\{path\ 6\}) = P(B).P(F|B).P(A|B).P(F|A) \quad (23)$$

$$Pr(\{path\ 7\}) = P(B).P(T|B).P(A|B).P(F|A) \quad (24)$$

$$Pr(\{path\ 8\}) = P(B).P(F|B).P(A|B).P(T|A) \quad (25)$$

$$Pr(\{path\ 9\}) = P(A).P(T|A).P(A|A).P(T|A) \quad (26)$$

$$Pr(\{path\ 10\}) = P(A).P(F|A).P(A|A).P(F|A) \quad (27)$$

$$Pr(\{path\ 11\}) = P(A).P(T|A).P(A|A).P(F|A) \quad (28)$$

$$Pr(\{path\ 12\}) = P(A).P(F|A).P(A|A).P(T|A) \quad (29)$$

$$Pr(\{path\ 13\}) = P(B).P(T|B).P(B|B).P(T|B) \quad (30)$$

$Pr(\{path\ 14\}) = P(B).P(F|B).P(B|B).P(F|B)$ (31)

$Pr(\{path\ 15\}) = P(B).P(T|B).P(B|B).P(F|B)$ (32)

$Pr(\{path\ 16\}) = P(B).P(F|B).P(B|B).P(T|B)$ (33)

In Fig (5), its illustrates all possible paths through the processes for two types of banknotes, A and B, with their respective states (T for not fake, F for fake), the model can be used to determine the path with the highest probability. This path represents the most likely sequence of events, indicating whether a banknote is fake or not. The probability of any given path in an HMM is determined by the product of the probabilities of each step in the path, including both the transition probabilities (moving from one state to another) and the emission probabilities (the probability of observing a certain outcome given a particular state). If you have a series of states and observations, the probability of a specific path in the HMM is calculated as follows:

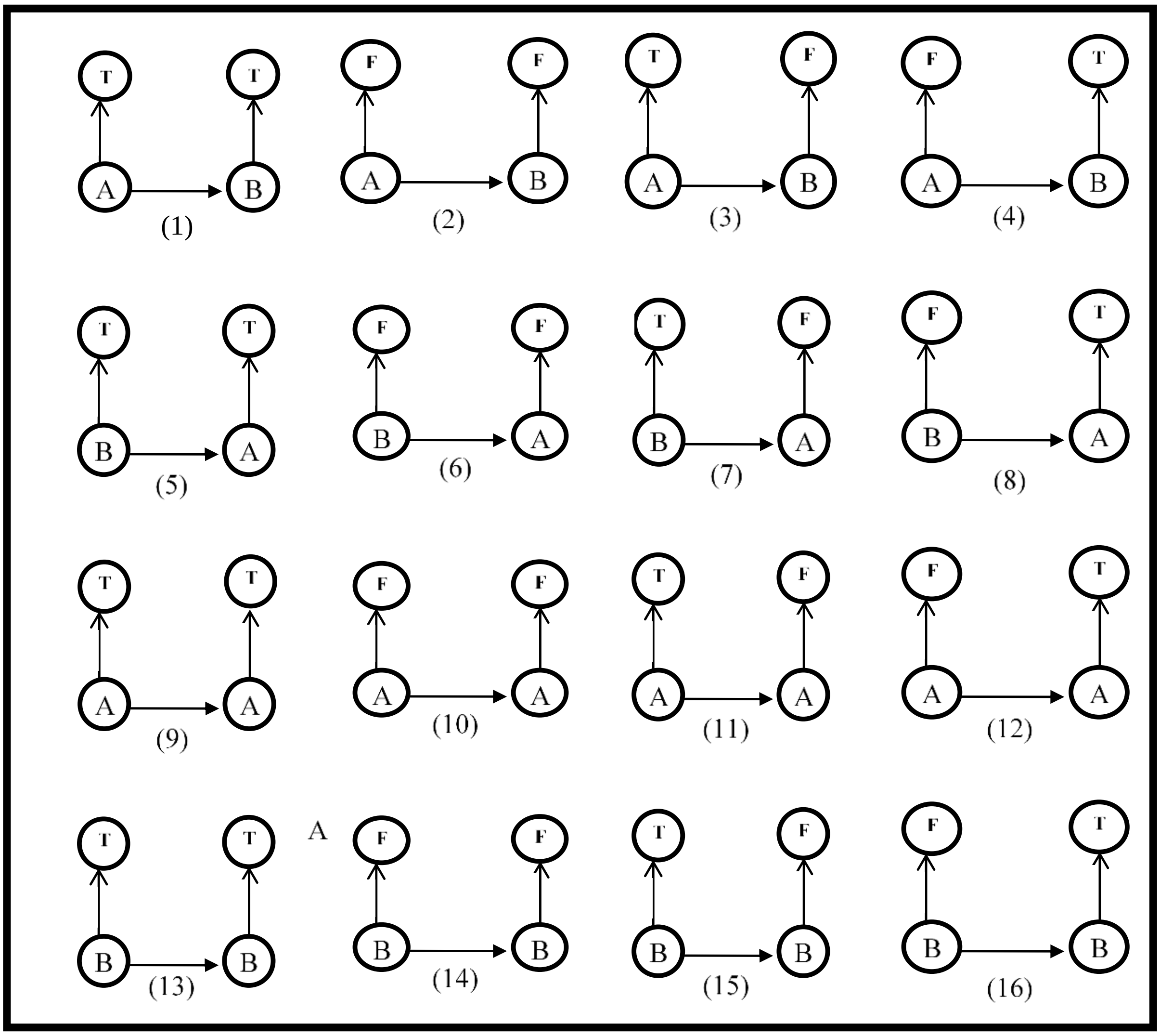
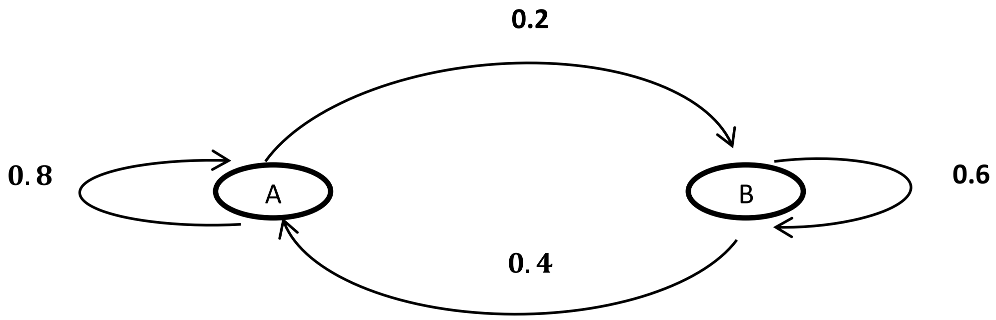


Fig (5):All paths of the two values "A" and "B" of the banknotes with their two different states "T" or "F"

4. APPLICATION ON REAL DATA.

To generate values between (0) and (1) they obtained using the Excel program by the code (= RAND () *(b-a)+a, a=0 and b=1). Or using Mathematica by the code (Table [Random [Real, {a, b}], {c}]), where c is the number of required probabilities.
Applying the previous code in Excel, then the following probabilities were obtained.
$P(A|A)$ = 0.8, $P(B|A) = 1$- $P(A|A)$= 0.2
$P(B|B)$ = 0.6, $P(A|B) = 1$- $P(B|B)$= 0.4
$P(A) = \frac{2}{3}$, $P(B) = \frac{1}{3}$

Application (1): Predicting the probabilities values of the values A and B of banknote using Markov model



Fig(6): Transition states probabilities of the values A and B of the banknotes using Markov model

Fig (6) shown the probabilities values of the transition states of the value A and B of banknotes using Markov model:

$$a_{ij}=\begin{bmatrix} P(A|A) & P(A|B) \\ P(B|A) & P(B|B) \end{bmatrix} = \begin{bmatrix} 0.8 & 0.4 \\ 0.2 & 0.6 \end{bmatrix} \tag{34}$$

And the initial state matrix is

$$\pi_i=\begin{bmatrix} \frac{2}{3} \\ \frac{1}{3} \end{bmatrix} \tag{35}$$

To obtain the probabilities of the cases of the type of currency in the deposit, it is as follows:

$$\text{N}=a_{ij}*\pi_i=\begin{bmatrix} 0.8 & 0.4 \\ 0.2 & 0.6 \end{bmatrix}\begin{bmatrix} \frac{2}{3} \\ \frac{1}{3} \end{bmatrix} = \begin{bmatrix} 0.668 \\ 0.332 \end{bmatrix} \tag{36}$$

Then $P(A) = 0.668$, $P(B) =$ 0.332

Application (2): Predicting the probabilities values of the types T and F of banknotes by Markov model

Similarly, for the states of two banknotes fake (F) or not fake (T)

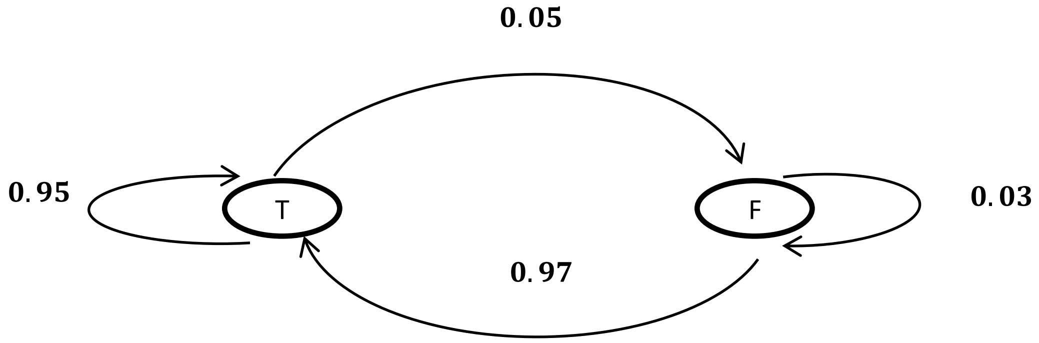


Fig (7): Transition states probabilities of the states F and T using the Markov model

$P(T)$ = 0.99, $P(F)$= 0.01,$P(T|F)$ = 0.97, $P(T|T)$ = 0.95, $P(F|T)$= 0.05, $P(F|F)$= 0.03

Fig (7) shows the probability values of the transition states of the types F and T of banknotes using the Markov model.

$$K=\begin{bmatrix}0.95 & 0.05\\ 0.97 & 0.03\end{bmatrix} \tag{37}$$

$$I=\begin{bmatrix}0.99\\ 0.01\end{bmatrix} \tag{38}$$

To obtain the probabilities of transition states of the value of the banknote after a limited number of deposits. Assuming that the number of deposits is 3.

$$a_{ij}^{(3)}=\begin{bmatrix}0.8 & 0.4\\ 0.2 & 0.6\end{bmatrix}\cdot\begin{bmatrix}0.8 & 0.4\\ 0.2 & 0.6\end{bmatrix}\cdot\begin{bmatrix}0.8 & 0.4\\ 0.2 & 0.6\end{bmatrix}=\begin{bmatrix}0.688 & 0.624\\ 0.312 & 0.376\end{bmatrix} \tag{36}$$

Therefore, the probabilities of moving after three deposits are as follows:

$P(A|A) =$ 0.688

$P(B|B)$ = 0.376

$P(A|B) = 0.312$

$P(B|A) = 0.624$

Application (3): Predicting the probabilities of different paths in HMM

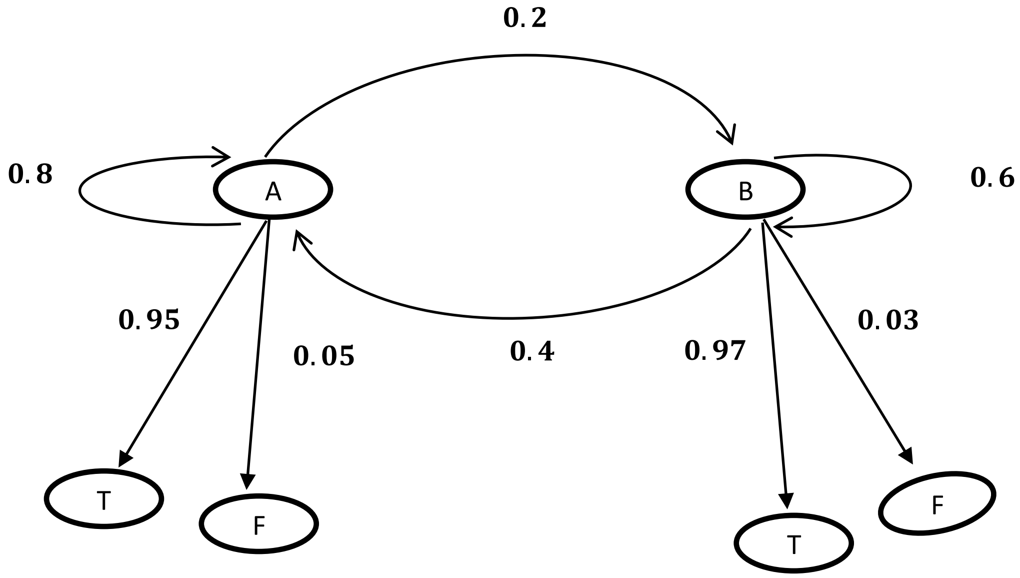


Fig. (8): Hidden Markov model of the two states of banknotes

Fig (8) represent the hidden Markov model of two banknotes A and B combined with their states Fake (F) or not Fake (T).Where N is the number of hidden states A and B , M represent the state F and T. the symbol $\{a_{ij}\}$ refer to the transition matrix and the symbol $\{b_{jk}\}$ is the observation matrix , while π_i is the initial state matrix.

Table (1): The probabilities of different paths for the states of the banknotes, fake or not fake

i	$Pr(\{path\ \mathrm{i}\})$	Probability value
1	$Pr(\{path\ 1\}) = P(A).P(T\|A).P(B\|A).P(T\|B)$	0.123481
2	$Pr(\{path\ 2\}) = P(A).P(F\|A).P(B\|A).P(F\|B)$	0.000201
3	$Pr(\{path\ 3\}) = P(A).P(T\|A).P(B\|A).P(F\|B)$	0.003819
4	$Pr(\{path\ 4\}) = P(A).P(F\|A).P(B\|A).P(T\|B)$	0.006499
5	$Pr(\{path\ 5\}) = P(B).P(T\|B).P(A\|B).P(T\|A)$	0.121638
6	$Pr(\{path\ 6\}) = P(B).P(F\|B).P(A\|B).P(F\|A)$	0.000198
7	$Pr(\{path\ 7\}) = P(B).P(T\|B).P(A\|B).P(F\|A)$	0.006402
8	$Pr(\{path\ 8\}) = P(B).P(F\|B).P(A\|B).P(T\|A)$	0.0035739
9	$Pr(\{path\ 9\}) = P(A).P(T\|A).P(A\|A).P(T\|A)$	**0.48374**
10	$Pr(\{path\ 10\}) = P(A).P(F\|A).P(A\|A).P(F\|A)$	0.00134
11	$Pr(\{path\ 11\}) = P(A).P(T\|A).P(A\|A).P(F\|A)$	0.02546
12	$Pr(\{path\ 12\}) = P(A).P(F\|A).P(A\|A).P(T\|A)$	0.02546
13	$Pr(\{path\ 13\}) = P(B).P(T\|B).P(B\|B).P(T\|B)$	0.1862982
14	$Pr(\{path\ 14\}) = P(B).P(F\|B).P(B\|B).P(F\|B)$	0.0001782
15	$Pr(\{path\ 15\}) = P(B).P(T\|B).P(B\|B).P(F\|B)$	0.0057618
16	$Pr(\{path\ 16\}) = P(B).P(F\|B).P(B\|B).P(T\|B)$	0.0059499

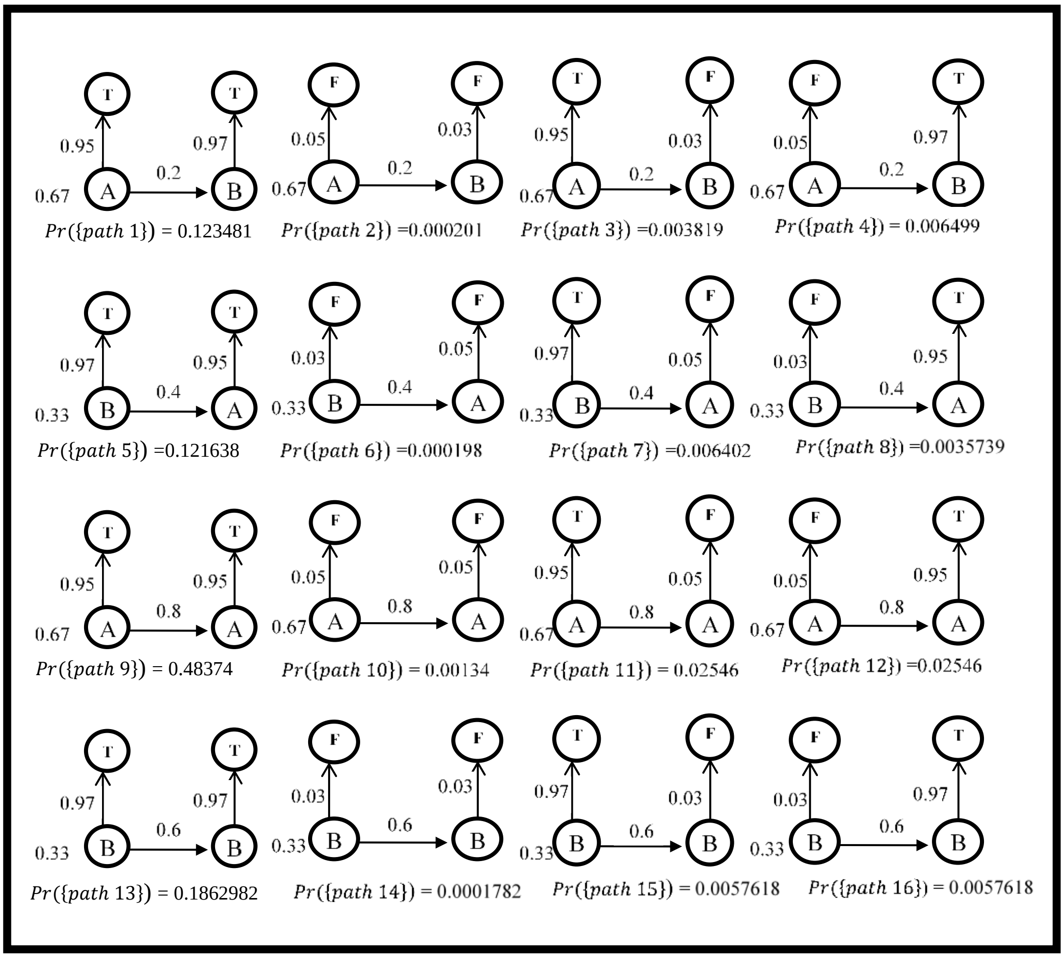


Fig. (9): All probability values of all paths for the states of banknotes

The data set of all probabilities in HMM is given in Fig (9). Table (1) shows the path number (i), its formula, and its probability value where i=1, 2,….,16. Consequently, in Fig(10) it was found that the most frequently occurring event is the most probable path 9, when the number of deposits is twice.

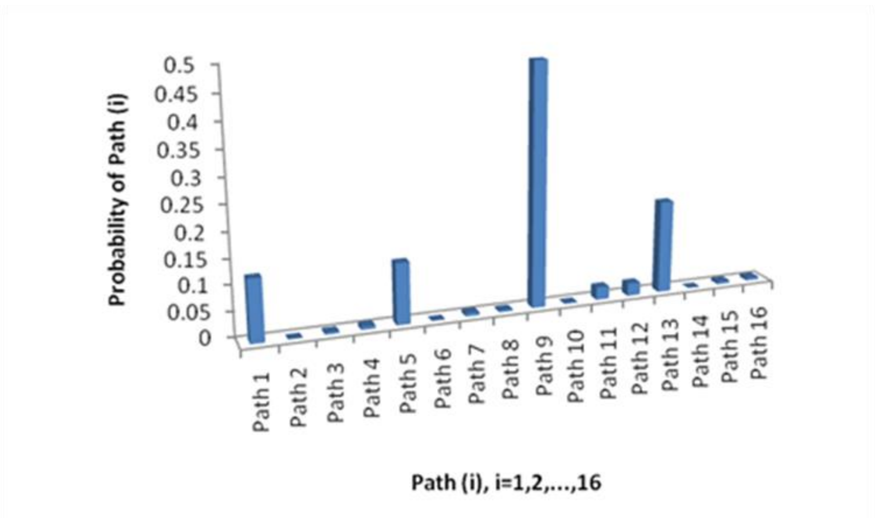


Fig. (10): All probability values for all paths of the banknotes states using hidden Markov model

5. CONCLUSION

This study discovered the probabilities of banknote values and their authenticity during deposits using Markov models. It also utilized hidden Markov models to predict the likelihood of observing banknote state trajectories upon deposit. The findings provide valuable insights into the dynamics of banknote transactions, offering a foundation for predictive modeling in various datasets. Researchers can extrapolate this study's methodologies to enhance fraud detection and risk assessment in financial transactions. By applying these models to other datasets, future trends in banknote values and authenticity can be predicted with greater accuracy, aiding in the development of robust financial systems.

6. APPENDIX

The following algorithm was written using the R program. That is to obtain the numerical results using the Markov model and the hidden Markov model to predict the values of banknotes and whether if they are fake or not.

Application (1):

```
>install.packages("markovchain")
       >libraray(markovchain)
       >banknoteStates<-c("typeA", "typeB")
       >banknoteMatrix<-matrix(data = c(0.80, 0.20, 0.40,
       +                    0.6), byrow = TRUE, nrow = 2,
       + dimnames = list(banknoteStates, banknoteStates))
       >print(banknoteMatrix)
             typeA   typeB
       typeA   0.8   0.2
       typeB   0.4   0.6
       >mcbanknote<-new("markovchain",states= c("typeA", "typeB"),
       + transitionmatrix=matrix(data=c(0.80, 0.20, 0.40,
       +                  0.6), byrow = byRow, nrow = 2,
       +                  name("markovchain"))
       >initialState<- c(0.66,0.33)
       >after2Deposit <- initialState * (banknoteMatrix * banknoteMatrix)
       >print(after2Deposit)
              typeA  typeB
       typeA  0.66  0.33
       >after7Deposit <- initialState * (banknoteMatrix ^7)
       >print(after7Deposit)
              typeA       typeB
       typeA 0.2097152  1.28e-05
```

Application (2):

```
>install.packages("markovchain")
       >libraray(markovchain)
       >banknotetype<-c("notfake", "fake")
```

```
>byRow<- TRUE
>banknoteMatrix<-matrix(data = c(0.95, 0.05, 0.97,
+                      0.03), byrow = byRow, nrow = 2,
+ dimnames = list(banknotetype, banknotetype))
>print(banknoteMatrix)
       Notfake fake
notfake 0.95   0.05
fake    0.97   0.03
>mcbanknote<-new("markovchain",states= c("notFake","Fake"),
+ transitionmatrix=matrix(data=c(0.95, 0.05, 0.0.97,
+                    0.03), byrow = byRow, nrow = 2,
+                    name("markovchain"))
>initialState<- c(0.99,0.01)
>after2Deposit <- initialState * (banknoteMatrix * banknoteMatrix)
>print(after2Deposit)
Notfake  fake
```

0.951 **0.0412**

```
>after7Deposit <- initialState * (banknoteMatrix ^7)
>print(after7Deposit)
Notfake   fake
```

0.9502 **0.0498**

Application (3):

```
>install.packages('HMM')
>library(HMM)
>state.set<- c("typeA","typeB")
>observation.set<- c("notFake","Fake")
>transition.matrix<-matrix(c(0.8, 0.2,0.4,0.6),2)
>observation.matrix<-matrix(c(0.95, 0.05,0.97,0.03),2)
>hmm=initHMM(state.set,observation.set,transProbs = transition.matrix,
startProbs =c(1,0),emissionProbs =  observation.matrix)
>Print(hmm)
$States
[1] "typeA" "typeB"
$Symbols
[1] "notFake" "Fake"
$startProbs
typeA    typeB
```

```
    1     0
$transProbs
       to
from   typeA typeB
typeA   0.8   0.4
typeB   0.2   0.6
$emissionProbs
       symbols
states  notFake Fake
typeA    0.95 0.97
typeB    0.05 0.03
# sequence of observations
>observations=c("notFake","Fake","notFake","notFake","notFake","Fake","Fake")
>viterbi(hmm,observations)
[1] "typeA" "typeA" "typeA" "typeA" "typeA" "typeA" "typeA"
```

7. REFERENCES

[1] A. B. Centeno, O. R. Terrades, J. L. Canet, and C. C. Morales, "Identity Document and Banknote Security Forensics: A Survey," 2019, [Online]. Available: http://arxiv.org/abs/1910.08993.

[2] Wang, "Anti-Counterfeiting for Polymer Banknotes Based on Polymer Substrate Fingerprinting," IEEE Trans. Inf. Forensics Secur., vol. 16, pp. 2823–2835, 2021, doi: 10.1109/TIFS.2021.3067440.

[3] B. Geva and M. Muraj, "The Digitization of Money: Stablecoins and CBDC," Banking & Finance Law Review, vol. 40, no. 1, pp. 115-141, 2024.

[4] M. M. El Genidy and M. S. Beheary, "Forecasting Methods in Various Applications Using Algorithm of Estimation Regression Models and Converting Data Sets into Markov Model," Complexity, vol. 2022, 2022, doi: 10.1155/2022/2631939.

[5] M. Rahul, N. Kohli, R. Agarwal, and S. Mishra, "Facial Expression Recognition Using Geometric Features and Modified Hidden Markov Model," Int. J. Grid Util. Comput., vol. 10, no. 5, pp. 488–496, 2019, doi: 10.1504/IJGUC.2019.102018.

[6] Kasianova, Kseniia. "Detecting Money Laundering Using Hidden Markov Model." PhD diss., Tesis de maestría). University of Tartu, 2020.

[7] S. Rajeshwari, "Vision-Based Banknote Recognition System," Int. J. Res. Appl. Sci. Eng. Technol., vol. 10, pp. 857-860, 2022, doi: 10.22214/ijraset.2022.45139.

[8] J. W. Lee, C. Lee, D. S. Jeong, K. Lee, and E. C. Lee, "Multi-Currency Integrated Serial Number Recognition Model of Images Acquired by Banknote Counters," Sensors, vol. 22, no. 22, pp. 8612, 2022. doi: 10.3390/s22228612.

[9] Addarrazi, O. Zealouk, H. Satori, and K. Satori, "The HMM Based Amazigh Digits Audiovisual Speech Recognition System," Mathematical Statistician and Engineering Applications, vol. 71, no. 4, pp. 2261-2278, 2022.

[10] Candra, Henry and Sulaiman, Susan, "Recognizing Face Using the Combination of Singular Value Decomposition and Hidden Markov Model Algorithms," 2023, doi: 10.1007/978-981-99-0248-4_38.

[11] Hassanpour, H. and Masoumifarahabadi, Payam, "Using Hidden Markov Models for Paper Currency Recognition," Expert Syst. Appl., vol. 36, pp. 10105-10111, 2009, doi: 10.1016/j.eswa.2009.01.057.

[12] D. Xiaoling, W. Cao, H. Cheng, and T. Zhang, "Hidden Markov Model-Driven Speech Recognition for Power Dispatch," in *International Conference on Applications and Techniques in Cyber Intelligence*, pp. 760-768, Springer International Publishing, 2022.

[13] Kumari, Aradhana, "Hidden Markov Model for Gesture Recognition," 2022, doi: 10.4018/978-1-7998-9434-6.ch006.

[14] Z. Wang, C. Bi, S. You, and J. Yao, "Hidden Markov Model-Based Video Recognition for Sports," Advances in Mathematical Physics, vol. 2021, pp. 1-12, 2021.

[15] Zubi, Zakaria and Idris, Eman, "Arabic Dialects System Using Hidden Markov Models (HMMs)," WSEAS Trans. Comput., vol. 21, pp. 304-315, 2022, doi: 10.37394/23205.2022.21.37.

[16] Namasudra, Suyel, Dhamodharavadhani, S., and Rathipriya, R., "Nonlinear Neural Network Based Forecasting Model for Predicting COVID-19 Cases," Neural Process. Lett., 2021, pp. 1-21.

[17] E. Behrends, S. Emphasis, and R. Mixing, "Introduction to Markov Chains," vol. 191, 2013, doi: 10.1007/978-1-4614-6765-6_3.

[18] E. Bas, "An Introduction to Markov Chains," Basics Probab. Stoch. Process., pp. 179–198, 2019, doi: 10.1007/978-3-030-32323-3_12.

[19] B. E. Elbaghazaoui, M. Amnai, and Y. Fakhri, "Predicting the Next Word Using the Markov Chain Model According to Profiling Personality," J. Supercomput., vol. 79, no. 11, pp. 12126–12141, 2023, doi: 10.1007/s11227-023-05125-2.

[20] T. Xue and H. Liu, "Hidden Markov Model and Its Application in Human Activity Recognition and Fall Detection: A Review," 1 Background: Human Activity Recognition and Fall Detection (HAR and FD), pp. 1–7.

[21] B. Mor, S. Garhwal, and A. Kumar, "A Systematic Review of Hidden Markov Models and Their Applications," Arch. Comput. Methods Eng., vol. 28, no. 3, pp. 1429–1448, 2021, doi: 10.1007/s11831-020-09422-4.

[22] Deng, Qi and Söffker, Dirk, "A Review of HMM-Based Approaches of Driving Behaviors Recognition and Prediction," IEEE Trans. Intell. Veh., vol. 7, no. 1, pp. 21-31, 2021.

[23] E. Thrane and C. Talbot, "An Introduction to Bayesian Inference in Gravitational-Wave Astronomy: Parameter Estimation, Model Selection, and Hierarchical Models," Publ. Astron. Soc. Aust., vol. 36, 2019, doi: 10.1017/pasa.2019.2.