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Modeling of Linear Magnetosonic Waves of Two Ion Species In the Martian Magnetosphere

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ABSTRACT

The propagation of linear magnetosonic waves in a homogeneous collisionless magne- tized plasma composed of two positive ions and electrons is investigated. The dispersion relation is derived using linear analysis to describe the dynamics of the behavior of the magnetosonic wave. There are two propagating modes for every ion species. Magnetized mode at low wavenumbers and ion cyclotron mode at higher wavenumbers. Electrons also have two propagating modes: a Whistler mode at low wavenumbers and an electron cyclotron mode at higher wavenumbers. Also, a field mode is found to propagate at very high frequencies. The propagation of these localized structures against variations in the magnetic field and the ion number densities has been discussed. Variations in the mag- netic field affect Alfven mode, Whistler mode, and the ion cyclotron modes. While the variations of number density influence the Alfven mode, Whistler mode, and field mode. This model is applied on the magnetosonic waves propagating in the Marian magnetosphere.

Key Words:

electromagnetic waves; Magnetosonic waves; Mars ionosphere; Linear magne- tosonic waves.

1. INTRODUCTION

Plasma waves are the oscillations or disturbances that occur in a medium composed of charged particles. These waves propagate carrying energy through the plasma. They can have various modes and characteristics, depending on factors such as the plasma density, temperature, and magnetic field strength. They can be categorized into different modes, including electrostatic waves, electromagnetic waves, and hybrid waves . Langmuir Waves or Electron Plasma Waves are longitudinal waves [6], meaning that the particles oscillate parallel to the direction of wave propagation. They are generated by collective oscillations of electrons in plasma. Ion acoustic waves are also longitudinal waves but are driven by the collective motion of ions in a plasma. They involve the interaction between ions and the electrons they displace, creating a restoring force that propagates the wave. However, ion cyclotron waves are electromagnetic waves that occur in the presence of a magnetic field and are associated with the motion of ions in a circular path around the field lines. They have a frequency close to the ion cyclotron frequency. The upper hybrid Waves are a combination of Langmuir waves and electromagnetic waves. They occur when the plasma frequency is close to the electron cyclotron frequency. However, the lower hybrid waves are a combination of Langmuir waves and ion acoustic waves. They occur when the plasma frequency is close to the ion cyclotron frequency.Whistler waves are electromagnetic waves that propagate along magnetic field lines and are often observed in magnetized plasmas. Alfvén waves are transverse waves that propagate along magnetic field lines in a plasma. They are driven by the interaction between magnetic fields and charged particles. These waves play a crucial role in many natural and laboratory plasma phenomena. They are involved in processes such as energy transfer, particle acceleration, and wave-particle interactions. Magnetosonic waves are a type of plasma wave that propagates in a magnetized plasma. They are a combination of magnetic and acoustic waves and exhibit both compressional and transverse motion [6, 4].

A lot of studies have been done on magnetosonic waves. Toward the linear magnetosonic waves; M. Toida, Y. Ohsawa, and T. Jyounouchi have investigated the propagating modes in the linear magnetosonic waves and shown that the resulting modes are slow and fast modes [11]. But for the nonlinear magnetosonic waves; Maruyama, K., Bessho, N., & Ohsawa have studied the interactions of non-thermal energetic ions with nonlinear magnetosonic waves by means of a one-dimensional (one space coordinate and three velocity components), relativistic, electromagnetic particle simulation code with full ion and electron dynamics [1]. Mushtaq, A., & Shah, H. A. have discussed obliquely propagating magnetosonic waves in an external magnetic field in electron-ion-positron plasma and discussed the linear approximation for the fast and the slow modes [2]. In a two-ion-species plasma, the magnetosonic wave is divided into two modes; the low-frequency mode and the high-frequency mode [9, 10]. The low-frequency mode is of the ion-ion hybrid resonance frequency, while the high-frequency mode is of the lower hybrid resonance frequency [5]. Recently, Masood, W., Shah, H. A., Mushtaq, A., & Salimullah, M. have investigated the linear and nonlinear properties of two-dimensional oblique propagation of dust magnetosonic waves [3].

The study of magnetohydrodynamics (MHD) plays a crucial role in understanding the behavior of plasmas in various astrophysical and laboratory environments. In particular, investigating linear magnetosonic waves has proven to be a fundamental aspect of MHD research. By considering the presence of multiple ion species within a plasma, this paper aims to present a comprehensive model that accurately describes the linear magnetosonic wave propagation in such systems. This work is organized as follows; the following section (2), explains the mathematical model. Section (3) discusses the numerical results and the parametric analysis of the propagating modes by changing the magnetic field B and the fluid density n.

2. Basic Equations

Assuming a linear magnetosonic wave propagates perpendicular to the magnetic field through a compressible, collisionless homogeneous plasma medium. The medium is composed of two positive ions (a and b) and electrons e. The electric field $\boldsymbol{E}$, the magnetic field $\mathbf{B}$, and the wavevector $\mathbf{k}$ of the propagating magnetosonic waves are defined as follows, respectively,$\mathbf{E} = E_1\hat{x}$, $\mathbf{B} = (\mathrm{B}_0 + \mathrm{B}_1)\hat{\mathrm{z}}$, $\mathbf{k} = \mathrm{k}\hat{\mathrm{y}}$

, where the subscripts 0,1 are the unperturbed and perturbed quantities, respectively. The governing equations are the fluid equation of motion, which are defined as follows,

$$m_j n_j \frac{\partial u_j}{\partial t} = q_j n_j \left(E + u_j \times B\right), \tag{1}$$

where ^{j} stands for j^{th} species(ions and electrons), $j = a, b$ and $e.m_j$ and q_j are the masses and the charges of the species j. n_j and u_j represent the density and the velocity of the j^{th} species, respectively. The right-hand side is the Lorentz force. Equation (1) is coupled with the following Maxwell's equations.

$$\nabla \times E = \quad \frac{\partial B}{\partial t}, \tag{2}$$

$$\nabla \times B = \mu_0 J + \mu_0 \ {}_0 \frac{\partial E}{\partial t}, \tag{3}$$

where ε_0 and μ_0 are the electric permittivity and the magnetic permeability of free space. The first term in Eq. (3) is $\mu_o \mathrm{J} = \mu_o \sum_j q_j n_{j0} u_{j1}$ which represents the current density, while the second term $\mu_0 \ \varepsilon_0 \frac{\partial E}{\partial t}$ is the displacement current density. Equation (3) can be rewritten as

$$\nabla \times B = \mu_0 \sum_j q_j n_{j0} u_{j1} + \mu_0 \ \varepsilon_0 \frac{\partial E}{\partial t}. \tag{4}$$

The linearized forms of Eq. (1) , in x- and y-directions, are:

$$m_j n_{j0} \frac{\partial u_{xj1}}{\partial t} = q_j n_{j0} (E_{x1} + u_{y1} B_{z0}) \tag{5}$$

$$m_j n_{j0} \frac{\partial u_{xj1}}{\partial t} = -q_j n_{j0} u_{x1} B_{z0} \tag{6}$$

The linear analysis is assumed in one dimension, where k is in the y-direction. The dependent variables are adopted as

$$\mathrm{n}_j = \mathrm{n}_{j0} + \mathrm{n}_{j1} e^{i(ky \quad \omega t)}, \tag{7}$$

$$B = B_0 z + B_1 e^{i(ky \quad \omega t)} z, \tag{8}$$

$$E = E_1 e^{i(ky \quad \omega t)} x, \tag{9}$$

$$J = J_1 e^{i(ky \quad \omega t)} x. \tag{10}$$

Where ω is the frequency of the propagating wave, while k is the wavenumber of this wave. Taking the curl of Eq. (2), then substituting into Eq. (4), we get

$$\left(\omega^2 - c^2 k^2\right) \quad E = - \quad \frac{i\omega}{\varepsilon_0} \sum_j q_j n_{j0} u_{j1}. \tag{11}$$

Linearizing Eq. (11), we get

$$\left(\omega^2 - c^2 k^2\right) \quad \mathrm{E}_x = - \quad \frac{i\omega}{\varepsilon_0} \sum_j q_j n_{j0} \mathrm{u}_{xj1}. \tag{12}$$

From Eqs.(5) and (6), we get the following expression for the species' velocity in x-direction

$$u_{xj1} = \frac{ie}{\omega m_j \left(1 - \frac{\omega_{cj}^2}{\omega^2}\right)} E_x, \tag{13}$$

Substituting Eq. (13) into Eq. (12), we obtain the linear dispersion relation as:

$$\omega^2 - c^2k^2 - \left(\sum_j \frac{\Omega_{pj}^2 \omega^2}{\omega^2 - \omega_{cj}^2}\right) = 0, \tag{14}$$

where Ω_{pj} is the plasma frequency of the j^{th} species which is defined as $\Omega_{pj} = \left(e^2 n_j / \varepsilon_0 m_j\right)^{1/2}$ and ω_{cj} is the j^{th} species cyclotron frequency that is given by $\omega_{cj} = eB/m_j$. Reformulating Eq (14) into a polynomial form, we get

$$C_8\omega^8 + C_6\omega^6 + C_4\omega^4 + C_2\omega^2 + C_0 = 0, \tag{15}$$

where $C_0, C_2, ..., C_8$ are coefficients of the ω and they are defined as follow,

$$C_8 = 1$$

$$C_6 = \left(\omega_{c(O_2)}^2 + \omega_{c(O)}^2 + \omega_{c(e)}^2 + \Omega_{p(O_2)}^2 + \Omega_{p(O)}^2 + +\Omega_{p(e)}^2 + c^2k^2\right)$$

$$C_4 = \omega_{c(O_2)}^2\omega_{c(O)}^2 + \omega_{c(O_2)}^2\omega_{c(e)}^2 + \omega_{c(O)}^2\omega_{c(e)}^2 + \Omega_{p(O_2)}^2\left(\omega_{c(O)}^2 + \omega_{c(e)}^2\right)$$

$$+ \ \Omega_{p(O)}^2\left(\omega_{c(O_2)}^2 + \omega_{c(e)}^2\right) + \Omega_{p(e)}^2\left(\omega_{c(O_2)}^2 + \omega_{c(O)}^2\right) + c^2k^2\left(\omega_{c(O_2)}^2 + \omega_{c(O)}^2 + \omega_{c(e)}^2\right)$$

$$C_2 = -\omega_{c(O_2)}^2\omega_{c(O)}^2\omega_{c(e)}^2 - \Omega_{p(O_2)}^2\omega_{c(O)}^2\omega_{c(e)}^2 - \Omega_{p(O)}^2\omega_{c(O_2)}^2\omega_{c(e)}^2 - \Omega_{p(e)}^2\omega_{c(O_2)}^2\omega_{c(o)}^2$$

$$- \ c^2k^2\left(\omega_{c(O_2)}^2\omega_{c(O)} + \omega_{c(O_2)}^2\omega_{c(e)}^2 + \omega_{c(O)}^2\omega_{c(e)}^2\right)$$

$$C_0 = c^2k^2\left(\omega_{c(O_2)}^2\omega_{c(O)}\omega_{c(e)}^2\right)$$

3. Numerical Solution and Discusion

In this work, we study the linear magnetosonic waves propagating in a two-ion-plasma medium. This model could be applied to the observed linear magnetosonic waves on Mars [7]. Linear magnetosonic waves have been observed in the Martian magnetosphere that propagate in the dayside ionosphere at an altitude of about 535.7 km and solar zenith angle (SZA) of about 77.8 [8] The medium is composed of two ions; oxygen O^+ , di-oxygen O_2^+ and electrons e^-. At the region of observation, the magnetic field strength $B \approx 22\text{nT}$, and the densities of the ions are; $n_{O^+} = (10 - 20)\times 10^6 \text{m}^{-3}$,

$n_{O_2^+} = (20 - 60)\times 10^6 \text{m}^{-3}$.

Solving the polynomial Eq.(15) gives 8 roots; half of them propagate in the forward (+ve) direction, and the other half propagates in the backward (-ve) direction, which means that we have a mirror mode comes from the magnetic field. Each root indicates a special mode in the plasma.

Figure (1) depicts the propagating modes in our medium. These modes depend on the type of the ions, which will be cleared below.

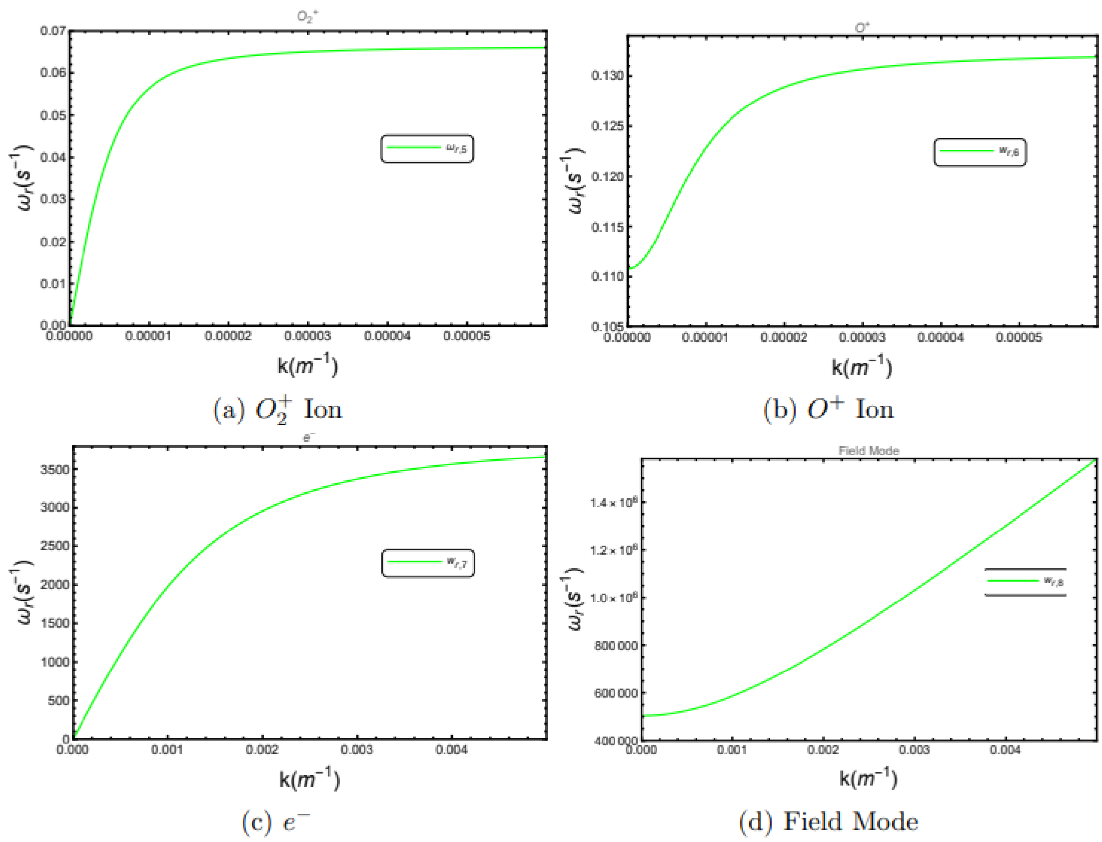


Figure 1: Depicts the propagating modes for each root: (a) O_2^+ species, (b) O^+ species, (c) e^- species, (d) field mode. At the upper ionosphere of Mars, which has the magnetic field $B \approx 22$ nT and the number density for each species are $n_{O_2^+} = 60\times 10^6\ m^{-3}$ and $n_{O^+} = 20\times 10^6\ m^{-3}$.

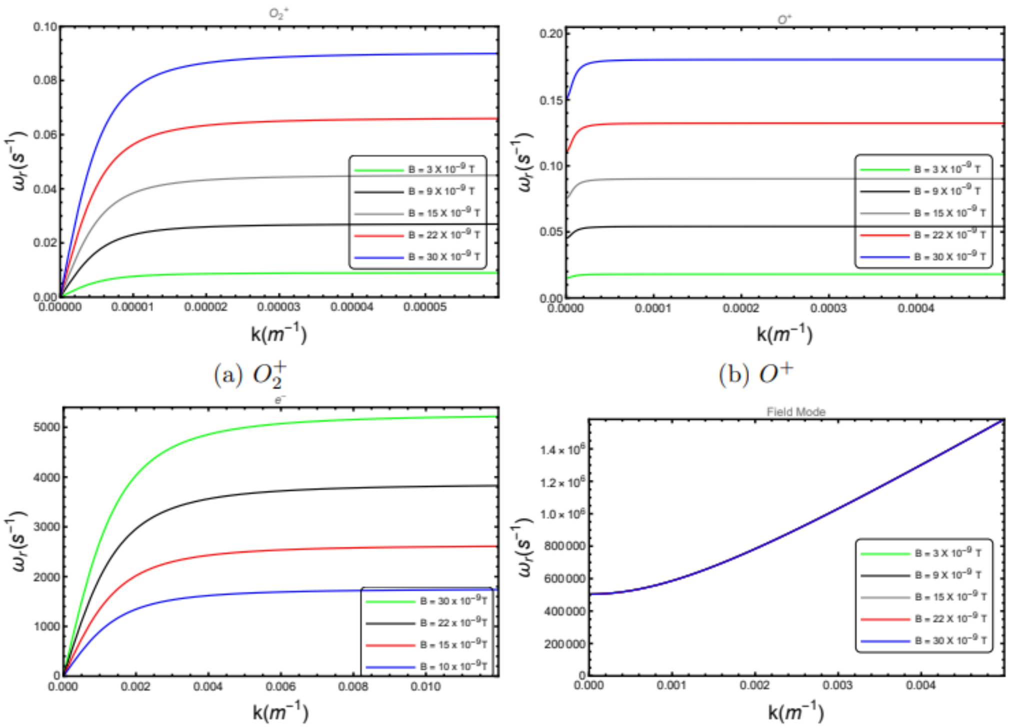
a. O_2^+ ion modes From Fig (1a), we see that the propagating modes produced by O_2^+ ion fluid are divided into two parts. A mode that propagates at low k ($k = (0 \quad 3)\times 10\ ^{5}\mathrm{m}\ ^{1}$) until it reaches k_c (k_c is the wavenumber at which the magnetized mode disappears and the propagating mode becomes ion cyclotron mode for the higher wavenumber k). The mode with $k < k_c$ is Alfven mode. We found that the propagating mode satisfies this dispersion relation proving that this is an Alfven mode. Then, as the wavenumber k increases the propagating mode saturates at the O_2^+ ion cyclotron frequency which means that the O_2^+ ion cyclotron mode becomes dominant after k_c. This mode propagates at the cyclotron frequency of the O_2^+ ion, i.e at $\omega_{c(o_2)} = 0.06629\ Hz$.

b. O^+ and e^- species modes
From Figs. (1b, 1c), we notice that the propagating modes in each ion fluid are also divided into two parts. At low $k < k_c$ the propagating mode is a Whistler mode. Increasing the wavenumber

$k > k_c$, each ion fluid has a propagating wave at the ion cyclotron frequency. Hence, it is called the ion cyclotron frequency mode. Each species saturates at the following frequencies; $\omega_{c(o_2)} = 0.06629 \quad Hz$, $\omega_{c(o)} = 1.32331 \quad Hz$, $\omega_{c(e)} = 3863.89 \quad Hz$.

c. The field mode

Fig (1d) indicates that the propagating mode is the magnetized analog of Langmuir mode.



(a) O_2^+ (b) O^+

(c) e^- (d) *field mode*

Figure 2: *shows the influence of varying the magnetic field on the propagating modes for each root; (a) O_2^+ species, (b) O^+ species, (c) e^- species, (d) field mode. At the upper ionosphere of Mars which has the number density for each species are* $n_{O_2^+} = 60 \times 10^6 \ m^{-3}$ *and* $n_{O+} = 20 \times 10^6 \ m^{-3}$

As the plasma parameters can affect the behavior of the existing modes. Here, we study the effects of each plasma parameter (the magnetic field B, and the density n_j) on the propagating modes for each species. Figure (2) introduces the effects of varying the magnetic field on the propagating waves on different ion fluids. It is noticed that varying the magnetic field B results in shiftiness of the saturation frequency (i.e. the ion cyclotron frequency) of the propagating wave, and of the critical wavenumber k_c after which the ion cyclotron mode becomes dominant. Verifying that decreasing the magnetic field B turns the wave into an ion cyclotron wave, i.e. disappearing the other propagating magnetized mode (Alfven or Whistler mode). Here, we study in detail the effect of changing the magnetic field B on each ion fluid as follows,

a. O_2^+ ion mode

From Fig (2a), we see that increasing the magnetic field gives similar propagating modes that are observed at higher frequencies because of the increase of the saturation frequency (carbondioxide cyclotron frequency $\omega_{c(O_2)}$). That's why, we observe that the saturation mode (ion cyclotron mode) becomes noticeable at higher frequencies by increasing B. Also, the magnetized mode (Alfven mode) lasts for higher wavenumber k, i.e. at lower wavelengths. Therefore, we can say that the Alfven mode propagates at smaller wavelengths λ and could last for higher frequencies by increasing the magnetic field.

b. O^+ and e^- species modes

From Figs (2b) and (2c), it is noticed that the saturation frequency (ion cyclotron frequency) occurs at higher ω_c by increasing the magnetic field B. The propagating Whistler mode lasts for higher frequency bands and larger wavenumber k. Hence, we can say that the Whistler mode disappears by decreasing the magnetic field B as expected. This effect is the same for oxygen ions O^+ and electrons e^-.

c. The field mode

This mode results from the displacement current which depends on the densities of the ions. That's why, it isn't affected by changing the magnetic field B at all. From Fig (2d), we see that the magnetized analog of Langmuir mode doesn't suffer any changes by varying the magnetic field B.

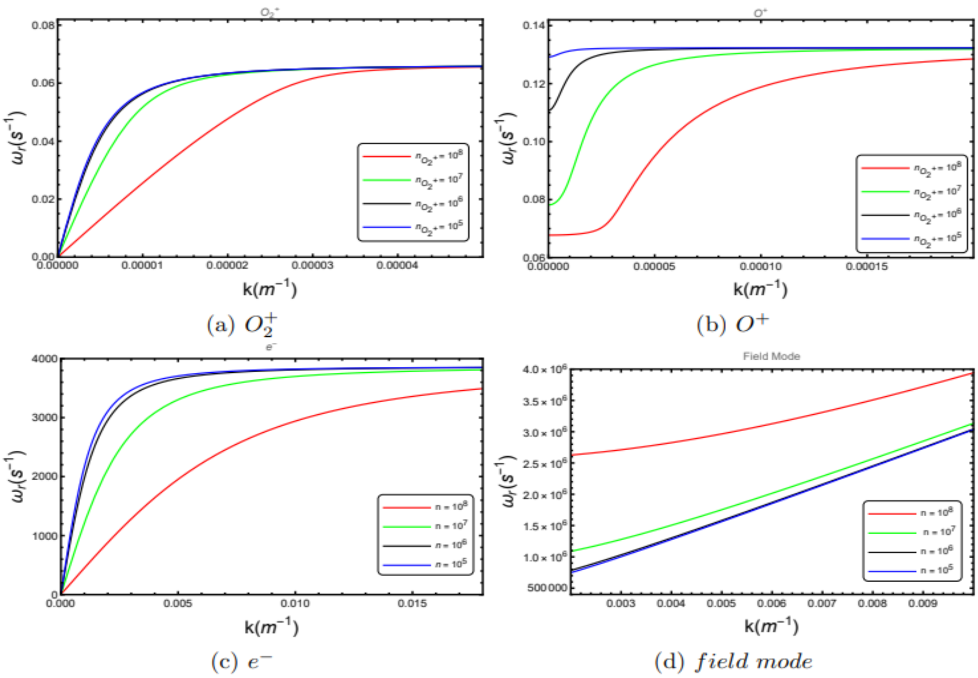


Figure 3: *shows the influence of varying the number density on the propagating modes for each root; (a) O_2^+ species, (b) O^+ species, (c) e^- species, (d) field mode. At the upper ionosphere of Mars which has the magnetic field $B \approx 22nT$*

Figure (3) introduces the effects of varying the density of the fluid species on the propagating modes. This shows that the ion cyclotron frequency ω_{cj} , at which the ion cyclotron mode propagates, isn't affected by changing the density of the species j. It just influences the Alfven and the Whistler modes making them last for higher wavenumbers k.

a. O_2^+ ion

From Fig.(3a), we see that the saturation occurs at larger wavenumber k by increasing the density n_{O_2} (i.e, $k_c \propto n_j$). But for the ion cyclotron mode, it has nothing related to the density except for retarding its observation at higher k, i.e., smaller wavelength λ.

b. O^+ and e^- species modes

From Fig.(3b and 3c), we notice that the saturation frequency ω_{cj} is not affected by varying the density of the species n_j. The magnetized mode (Whistler mode) begins to be observed at higher wavenumber k and lasts for a greater frequency ω by increasing the density of the ion fluid n_j.

c. The field mode

But for the magnetized Langmuir mode, it's obvious from Fig (3d) that the waves propagate at larger frequencies by increasing the whole density of the plasma medium.

4. CONCLUSION REMAEKS

In this work, we have carried out a linear analysis of magnetosonic waves observed in the upper ionosphere of Mars. This medium is composed of two cold ion species; oxygen O^+ and di-oxygen O_2^+ and the neutralizing electrons e^-. Our analysis has shown that every ion fluid has propagating waves with two different modes; one of them is excited due to the magnetic field B. This mode for the ion species O_2^+ is an Alfven mode. While for the other species, it's a Whistler mode. Both modes appear only when there is a magnetic field B. These magnetized modes only propagate at low wavenumber $k < k_c$. At larger wavenumbers $k > k_c$, the dominant mode at all ion fluids, is the ion cyclotron mode. This mode propagates at the cyclotron frequency of each species ω_{cj}. We also have investigated the influence of different plasma parameters (magnetic field B, density n) on the propagating modes. It's shown that both parameters don't excite any new modes within the ion fluids. Varying the magnetic field, results in shiftiness of the cyclotron frequency and the critical wavenumber k_c after which the propagating mode becomes the ion cyclotron mode. Therefore, increasing the magnetic field B makes the magnetized modes propagate for higher ω and last for larger k. But for the density variations, we have seen that increasing the number density for each species results in an increase in the critical wavenumber k_c, i.e. increase in the frequency range of the propagating Alfven or Whistler-mode.

5. REFERENCES

[1] K. Maruyama, N. Bessho, and Y. Ohsawa, "Acceleration of energetic ions by nonlinear magnetosonic waves," Physics of Plasmas, vol. 5, no. 9, pp. 3257-3263, 1998.

[2] A. Mushtaq and H. A. Shah, "Effects of positron concentration, ion temperature, and plasma value on linear and nonlinear two-dimensional magnetosonic waves in electron- positron-ion plasmas," Physics of Plasmas, vol. 12, no. 1, pp. 012301-012301, 2005.

[3] W. Masood, H. A. Shah, A. Mushtaq, and M. Salimullah, "Linear and nonlinear properties of an obliquely propagating dust magnetosonic wave," Journal of Plasma Physics, vol. 75, no. 2,

pp. 217-233, 2009.

[4] J. A. Bittencourt, Fundamentals of Plasma Physics. Springer Science Business Media, 2004.

[5] S. J. Buchsbaum, ”Resonance in a plasma with two ion species,” The Physics of Fluids, vol. 3, no. 3, pp. 418-420, 1960.

[6] F. F. Chen, Introduction to Plasma Physics. Springer Science Business Media, 2012.

[7] C. M. Fowler, L. Andersson, R. E. Ergun, Y. Harada, T. Hara, G. Collinson, W. K. Peterson, J. Espley, J. Halekas, J. Mcfadden, et al., ”MAVEN observations of solar wind-driven magnetosonic waves heating the Martian dayside ionosphere,” Journal of Geophysical Research: Space Physics, vol. 123, no. 5, pp. 4129-4149, 2018.

[8] C. M. Fowler, K. G. Hanley, J. P. McFadden, C. C. Chaston, J. W. Bonnell, J. S. Halekas, J. R. Espley, G. A. DiBraccio, S. J. Schwartz, C. Mazelle, et al., ”MAVEN observations of low frequency steepened magnetosonic waves and associated heating of the Martian nightside ionosphere,” Journal of Geophysical Research: Space Physics, vol. 126, no. 10, pp. e2021JA029615, 2021.

[9] A. B. Mikhailovskii and A. I. Smolyakov, ”Zh. Eksp. Teor. Fiz,” vol. 88, pp. 189-204, 1985.

[10] U. Motschmann, K. Sauer, Th. Roatsch, and J. F. McKenzie, ”Journal of Geophysical Research: Space Physics,” vol. 96, no. A8, pp. 13841-13848, 1991.

[11] M. Toida, Y. Ohsawa, and T. Jyounouchi, ”Physics of Plasmas,” vol. 2, no. 9, pp. 3329-3334, 1995.