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An Accurate Method for Estimating the Parameters of the Generalized Extreme Value Distribution Using its Moments

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ABSTRACT

Wind speed is a clean energy source that generates electricity. Researchers in this field always need an accurate statistical model to give them high-precision statistical measurements to build power-generation electric systems. This study presents a mathematical model for the maximum wind speed (MWS) in Port Said city by determining the fitting probability distribution. The moment-generating function is used to estimate the Generalized Extreme Value Distribution (GEVD) parameters. The purpose is to obtain a single equation in a single parameter using average, variance, and median formulas of (GEVD), which can be solved numerically. The properties of the cumulative distribution function, method of maximum likelihood estimation (MLE), percentiles, quartiles, nonlinear regression, Anderson-Darling test, Kolmogorov-Smirnov (K-S) test, and Kruskal Wallis test are applied to assess the utility of the proposed distribution. The GEVD is compared with the three-parameter Weibull (W-3P) distribution and with other competing distributions. Finally, the GEVD is the best for modeling MWS data. The statistical measurements of MWS are derived with high accuracy. That will enable researchers to find the best estimation method of the distribution for the actual data.

Key Words:

Maximum wind speed, generalized extreme value distribution, three-parameter Weibull distribution, moment generating function, maximum likelihood estimation.

1. INTRODUCTION

The difference in air pressure levels causes air to move from places of high pressure to low-pressure areas. The lower the air pressure level difference, the higher the wind speed. Thus, temperature indirectly affects the MWS. The three- parameter GEVD, of which MWS modeling is an important application, is the most widely applied statistical distribution for climate modeling.

Furthermore, twenty years of monthly maximum temperature (MT) have been provided by the Cameroon Development Corporation (CDC) in Mbonge. Fitting this dataset into the GEV family of distributions, the three-parameters of the GEV model were discussed to compare the dataset with the Freshet, Weibull, and Gumbel models [1].

Remarkably, GEVD was used to model MTs, using a dataset obtained from the Penang weather station from 2000 to 2009. The parameters were estimated by applying the L-moments, the MLE methods, and the Kolmogorov-Smirnov and Anderson-Darling tests [2]. The average of MTs was recorded at twenty-two meteorological stations in Malaysia. Likewise, the annual MTs were modeled using the Mann-Kendall (MK) test and the stationary and non-stationary GEVD [3]. A previous study evaluated climate models to simulate rainfall and minimum temperature. They were based on the characteristics of the probability density function for each variable and each region [4]. A hybrid computational method for estimating solar radiation in Iran is presented and discussed to construct a homogeneous data set and study its evaluation. Experimental models were used to estimate solar radiation based on meteorological parameters [5, 6, 7]. In another study, the spatial distribution of solar radiation was estimated from daily temperature using the Bristow-Campbell model. An artificial neural network ensemble model was also created to estimate solar radiation from satellite images [8, 9].

In recent years, researchers have still used the estimation method Maximum Likelihood (ML) to estimate the parameters of the different distributions. Contrary to this, this study relies on converting the three parameters of the GEVD into one equation using the moments-generating function of the GEVD. Thus, it can be solved numerically to obtain an estimation of these parameters. After which, it is substituted in the statistics equations of GEV to derive the values of the other parameters. The optimal solutions to the estimations of the parameters were determined on the basis of satisfying the properties of the cumulative distribution function of GEV in addition to covering the domain of the MWS. The methodology in this study saves time and effort in solving the equations arising from the use of the MLE method. The results of estimating the parameters of GEV were accurate and with minimal estimation error. The Kolmogorov-Smirnov and Anderson-Darling tests, combined with the GEVD, analyze the actual dataset of the MWS in Port Said city. Moreover, the Kruskal-Wallis test is applied to accept the established distribution. The GEVD is more suitable than the W-3P and other competing distributions. Finally, validate the significant results of the study by comparing the values of percentiles and quartiles for both the generalized extreme value distribution and the actual dataset.

This paper is organized as follows: Section 2 contains the definition of MWS, the proposed dataset, software used in this research, the expressions of the pdf and cdf for the GEV and W-3P distributions, moment generating function, mean, variance, median, obtaining the method of MLE of parameters for the GEV and W-3P models, and comparing the GEVD with other rival models. Section 3 provides results and discussion to assess the efficiency of the proposed distributions and methods. Finally, the conclusion is discussed in Section 4.

2. MATERIALS AND METHODS

2.1 Maximum wind speed (MWS)

It is the rate at which air moves in a particular area or from high pressure to low pressure, usually due to changes in temperature, measured in meters per second (m/s) [10].

2.2 Dataset

The actual dataset of the MWS was measured during the year 2015 by the Department of physics, Faculty of science, Port-Said University, Egypt to assess the MWS in Port Said city, as shown in Fig. (1).

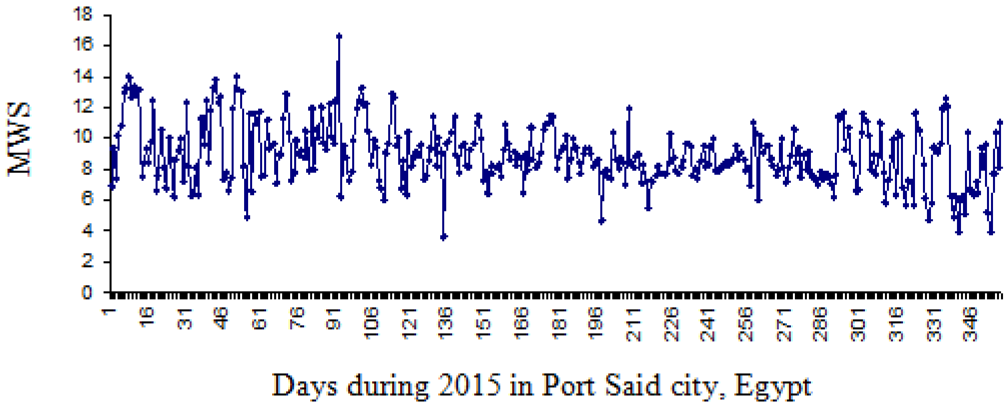

Fig. 1: Actual dataset of the MWS during year 2015 at Port Said, Egypt.

2.3 Software

The following programs have been performed on the dataset of MWS:

- EasyFit professional, version 5.5 (February 2010),MathWave Technologies, http://www.mathwave.com.
- Mathematica 8, version 8.0.1 (March 2011), Wolfram Mathematica, http://www.wolfram.com.
- SPSS 16.0 (IBM SPSS software, New York, USA).

2.4 Generalized extreme value distribution (GEVD)

Suppose that X is a continuous random variable representing the MWS and it has GEVD, which includes three types of distributions: type I-Gumbel ($\alpha_w = 0$), type II-Fréchet ($\alpha_w > 0$), and the type III-Weibull ($\alpha_w < 0$). The probability density function (pdf) is defined as:

$$f_{GEV}(x;\ \alpha_w, \theta_w, \lambda_w) = \frac{1}{\theta_w} g(x)^{\alpha_w+1}\, e^{-g(x)}, \qquad x \in \begin{cases} \left(-\infty, \lambda_w - \frac{\theta_w}{\alpha_w}\right] & if\ \alpha_w < 0, \\ (-\infty, \infty) & if\ \alpha_w = 0, \\ \left[\lambda_w - \frac{\theta_w}{\alpha_w}, \infty\right) & if\ \alpha_w > 0, \end{cases} \tag{1}$$

where,

$$g(x) = \begin{cases} \left(1 + \frac{\alpha_w}{\theta_w}(x - \lambda_w)\right)^{\frac{-1}{\alpha_w}} & if\ \alpha_w \neq 0, \\ e^{-\frac{(x-\lambda_w)}{\theta_w}} & if\ \alpha_w = 0. \end{cases} \tag{2}$$

The corresponding (cdf) $F_{GEV}(x;\ \alpha_w, \theta_w, \lambda_w)$ is given as

$$F_{GEV}(x;\ \alpha_w, \theta_w, \lambda_w) = e^{-g(x)} = \begin{cases} e^{-\left(1+\frac{\alpha_w}{\theta_w}(x-\lambda_w)\right)^{\frac{-1}{\alpha_w}}} & -\infty < x \le \lambda_w - \theta_w/\ \alpha_w \ \text{ for } \alpha_w < 0, \\ & \lambda_w - \theta_w/\ \alpha_w \le x < \infty \ \text{ for } \alpha_w > 0, \\ e^{-e^{-\frac{(x-\lambda_w)}{\theta_w}}} & -\infty < x < \infty \ \text{ for } \alpha_w = 0. \end{cases} \tag{3}$$

Such that $\alpha_w \in R,\ \theta_w > 0$ and $\lambda_w \in R$ are the shape, scale and location parameters, respectively [11].

2.5 Three-parameter Weibull distribution (W-3P)

Let X be a continuous random variable representing the MWS and it has W-3P distribution. Then, the probability density function (pdf) and cumulative distribution function (cdf) of the W-3P distribution are respectively given by

$$f_{W-3P}(x; \beta_w, \lambda_w, \alpha_w) = \begin{cases} \frac{\beta_w}{\lambda_w}\left(\frac{x-\alpha_w}{\lambda_w}\right)^{\beta_w - 1} e^{-\left(\frac{x-\alpha_w}{\lambda_w}\right)^{\beta_w}} & x > \alpha_w, \\ 0 & x \le \alpha_w. \end{cases} \tag{4}$$

$$F_{W-3P}(x; \beta_w, \lambda_w, \alpha_w) = \begin{cases} 1 - e^{-\left(\frac{x-\alpha_w}{\lambda_w}\right)^{\beta_w}} & x > \alpha_w, \\ 0 & x \le \alpha_w, \end{cases} \tag{5}$$

where, $\beta_w \ge 0$, $\lambda_w \ge 0$ and $\alpha_w \ge 0$ are the shape, scale and location parameters, respectively [12, 13].

2.6 Moment generating function

The moment generating function of the GEVD is defined as:

$$M(t) = E(e^{tx}) = \int_{-\infty}^{a} e^{tx} f(x)\, dx, \text{ where } a = \lambda_w - \frac{\theta_w}{\alpha_w} \text{ and } \alpha_w < 0$$

$$M(t) = e^{\left(\lambda_w - \frac{\theta_w}{\alpha_w}\right)t}\left[1 + \sum_{n=1}^{\infty} \frac{(\theta_w\ t)^n}{n!\ \alpha_w^n} \Gamma(1 - n\,\alpha_w)\right]. \tag{6}$$

Then, the first moment around zero is

$$Mean = M'(0) = -\frac{\theta_w}{\alpha_w} + \lambda_w + \frac{\theta_w\, \Gamma(1-\ \alpha_w)}{\alpha_w}. \tag{7}$$

The second moment around zero takes the following formula:

$$M''(0) = \left(-\frac{\theta_w}{\alpha_w} + \lambda_w\right)^2 + \frac{\theta_w^2\, \Gamma(1 - 2\,\alpha_w)}{\alpha_w^2} + \frac{2\,\theta_w\left(-\frac{\theta_w}{\alpha_w} + \lambda_w\right)\Gamma(1-\ \alpha_w)}{\alpha_w}. \tag{8}$$

Thus, from Eqs. (7) and (8)

$$Variance = M''(0) - \left(M'(0)\right)^2 = \frac{\theta_w^2\left(\Gamma(1 - 2\,\alpha_w) - \left(\Gamma(1-\ \alpha_w)\right)^2\right)}{\alpha_w^2}. \tag{9}$$

On the other hand, the median is obtained from Eq. (3) as follows:

$$Median = x = \lambda_w + \frac{\theta_w((\ln 2)^{-\alpha_w} - 1)}{\alpha_w}. \tag{10}$$

Similarly, the moment generating function of the W-3P distribution is shown as:

$$M(t) = E(e^{tx}) = e^{\alpha_w t} + \frac{\lambda_w}{\beta_w}\sum_{r=0}^{\infty}\frac{\alpha_w^r \ t^{r+1}}{r!}\sum_{n=0}^{r}\binom{r}{n}\left(\frac{\lambda_w}{\alpha_w}\right)^n \Gamma\left(\frac{n+1}{\beta_w}\right), \tag{11}$$

where, $\binom{r}{n} = \frac{r!}{n!\ (r-n)!}$ and $r = 0, 1, 2, \cdots$.

The mean and the variance are respectively written as

$$Mean = M'(0) = \alpha_w + \lambda_w \Gamma\left(1 + \frac{1}{\beta_w}\right). \tag{12}$$

$$Variance = \lambda_w^2 \left(\Gamma\left(1 + \frac{2}{\beta_w}\right) - \Gamma^2\left(1 + \frac{1}{\beta_w}\right)\right). \tag{13}$$

Then, the median is obtained from Eq. (5) as

$$Median = x = \alpha_w + \lambda_w (\ln 2)^{\frac{1}{\beta_w}}. \tag{14}$$

2.7 Maximum likelihood estimation (MLE) method

This section introduced the MLE method for the GEV and W-3P distributions.

The parameters estimation of the GEV and W-3P models are derived by utilizing the method of MLE. Let $X_1, X_2, X_3, \cdots, X_n$ be a random sample from GEV$(x;\ \alpha_w, \theta_w, \lambda_w)$ and W$(x; \beta_w, \lambda_w, \alpha_w)$, respectively. The likelihood functions are given by

$$L(x;\ \alpha_w, \theta_w, \lambda_w) = \prod_{i=1}^{n} f_{GEV}(x_i;\ \alpha_w, \theta_w, \lambda_w)$$

$$= \prod_{i=1}^{n}\frac{1}{\theta_w}\left(1 + \frac{\alpha_w}{\theta_w}(x_i - \lambda_w)\right)^{-\left(1+\frac{1}{\alpha_w}\right)} e^{-\left(1+\frac{\alpha_w}{\theta_w}(x_i-\lambda_w)\right)^{\frac{-1}{\alpha_w}}}.$$

Thus, the log-likelihood function is

$$\ell = -n \log\theta_w - \sum_{i=1}^{n}\left(\left(1 + \frac{1}{\alpha_w}\right)\log(z_i) + (z_i)^{\frac{-1}{\alpha_w}}\right), \tag{15}$$

where $\alpha_w \neq 0$ and $z_i = \left(1 + \frac{\alpha_w}{\theta_w}(x_i - \lambda_w)\right)$.

The following equations are formed by taking the derivatives of the previous equation with respect to the three parameters and equating it to zero

$$\frac{\partial \ell}{\partial \alpha_w} = \frac{1}{(\alpha_w)^2} \sum_{i=1}^{n} \left(\log(z_i) \left(1 - (z_i)^{\frac{-1}{\alpha_w}} \right) - \frac{\left(1 + \alpha_w - (z_i)^{\frac{-1}{\alpha_w}} \right)}{z_i} \left(\frac{\alpha_w}{\theta_w} (x_i - \lambda_w) \right) \right) = 0 \tag{16}$$

$$\frac{\partial \ell}{\partial \theta_w} = -\frac{n}{\theta_w} + \frac{1}{\theta_w} \sum_{i=1}^{n} \left(\frac{\left(1 + \alpha_w - (z_i)^{\frac{-1}{\alpha_w}} \right)}{z_i} \left(\frac{x_i - \lambda_w}{\theta_w} \right) \right) = 0 \tag{17}$$

$$\frac{\partial \ell}{\partial \lambda_w} = \frac{1}{\theta_w} \sum_{i=1}^{n} \left(\frac{\left(1 + \alpha_w - (z_i)^{\frac{-1}{\alpha_w}} \right)}{z_i} \right) = 0 \tag{18}$$

$$L(x; \beta_w, \lambda_w, \alpha_w) = \prod_{i=1}^{n} f_{W-3P}(x_i; \beta_w, \lambda_w, \alpha_w)$$

$$= \prod_{i=1}^{n} \frac{\beta_w}{\lambda_w} \left(\frac{x_i - \alpha_w}{\lambda_w} \right)^{\beta_w - 1} e^{-\left(\frac{x_i - \alpha_w}{\lambda_w} \right)^{\beta_w}}.$$

The log-likelihood function is

$$\ell = \sum_{i=1}^{n} \left(\log(\beta_w) - \beta_w \log(\lambda_w) + (\beta_w - 1) \log(x_i - \alpha_w) - \left(\frac{x_i - \alpha_w}{\lambda_w} \right)^{\beta_w} \right). \tag{19}$$

Then,

$$\frac{\partial \ell}{\partial \beta_w} = \sum_{i=1}^{n} \left(\frac{1}{\beta_w} + \log(x_i - \alpha_w) - \log(\lambda_w) - \left(\frac{x_i - \alpha_w}{\lambda_w} \right)^{\beta_w} \log\left(\frac{x_i - \alpha_w}{\lambda_w} \right) \right) = 0. \tag{20}$$

$$\frac{\partial \ell}{\partial \lambda_w} = \sum_{i=1}^{n} \left(-\frac{\beta_w}{\lambda_w} + \left(\frac{\beta_w}{\lambda_w} \right) \left(\frac{x_i - \alpha_w}{\lambda_w} \right)^{\beta_w} \right) = 0. \tag{21}$$

$$\frac{\partial \ell}{\partial \alpha_w} = \sum_{i=1}^{n} \left(-\frac{(\beta_w - 1)}{(x_i - \alpha_w)} + \left(\frac{\beta_w}{\lambda_w} \right) \left(\frac{x_i - \alpha_w}{\lambda_w} \right)^{\beta_w - 1} \right) = 0. \tag{22}$$

Multiple nonlinear equations are solved numerically to give estimates of the GEV and W-3P parameters.

3. RESULTS AND DISCUSSION

In this section, we begin by juxtaposing the relevant results of the significant previous studies with the results of the present study. Then, we focus on the points of contribution that, hopefully, foreground the significance of the present study.

Earlier research looked at the basic weather pattern and trend of daily MWS in northwest Europe and part of the North Atlantic using gamma distributions in a generalized linear model. Similarly, some important features of the regional daily MWS distributions and trends during the last few decades, linked to some large-scale changes, were revealed [14, 15]. One important study from the past used the Monte Carlo simulation to compare how well Generalized Maximum Likelihood (GML), Maximum Likelihood (ML), L moments (LM), and Method of Moments (MOM) worked with the Generalized Extreme Value (GEV) distribution and the Generalized Pareto (GP) function with a Poisson model. Bayesian prior distributions were used to restrict estimated values to a statistically or physically reasonable range in a generalized maximum-likelihood (GMLE) analysis. The GML quartile estimator employed a beta prior distribution, and it proved better than MOM [16, 17]. Nonetheless, different probability models were applied and employ the MLE method to analyze monthly average wind speed in South-East Nigeria. While, different estimation methods were utilized for models with variations in the fitting at different times and locations. Moreover, the joint effect of wind direction and wind speed in Shanghai Baoshan were discussed by applying the joint distribution probability and the wind-ice joint probability distribution were studied in Southwest China [18, 19, 20, 21].

This study relies on the moments of the three-parameter distribution function and transforms the problem of estimating three parameters into estimating one parameter in terms of the other parameters. Use the numerical solution of an equation for one parameter and the properties of the cumulative distribution function to determine optimal solutions for the estimated parameters.

Hence, the cdf ($F(x)$) is

$$F(x) = 0.25 \;\; if \;\; x = Q_{x_1} = 7.69 \tag{23}$$

$$F(x) = 0.50 \;\; if \;\; x = Q_{x_2} = 8.68 \tag{24}$$

$$F(x) = 0.75 \;\; if \;\; x = Q_{x_3} = 10.05 \tag{25}$$

where Q_{x_1}, Q_{x_2} and Q_{x_3} are the quartiles of the dataset of the MWS.

On the other hand, from Eqs. (7), (9) and (10) the mean, median, and variance are given by

$$Mean = \lambda_w + \theta_w \left(\frac{\Gamma(1 - \alpha_w) - 1}{\alpha_w} \right) = 8.9146 \tag{26}$$

$$Median = \lambda_w + \frac{\theta_w((\ln 2)^{-\alpha_w} - 1)}{\alpha_w} = 8.68 \tag{27}$$

$$Variance = \frac{\theta_w^2 \left(\Gamma(1-2\,\alpha_w) - \left(\Gamma(1-\,\alpha_w)\right)^2\right)}{\alpha_w^2} = 3.8637. \tag{28}$$

Hence, from Eqs. (26) and (27)

$$\theta_w \left[\frac{\Gamma(1-\,\alpha_w)-1}{\alpha_w} - \frac{(\ln 2)^{-\,\alpha_w}\,-1}{\alpha_w}\right] = 0.2346$$

$$\theta_w = \frac{0.2346\,\alpha_w}{(\Gamma(1-\,\alpha_w)-1) - ((\ln 2)^{-\,\alpha_w}\,-1)}$$

$$\theta_w = \frac{0.2346\,\alpha_w}{\Gamma(1-\,\alpha_w) - (\ln 2)^{-\,\alpha_w}}$$

$$\theta_w^2 \cong \frac{0.0550\,\alpha_w^2}{[\Gamma(1-\,\alpha_w) - (\ln 2)^{-\,\alpha_w}]^2}. \tag{29}$$

Substituting Eq. (29) in Eq. (28), then

$$\frac{\Gamma(1-2\,\alpha_w) - \left(\Gamma(1-\,\alpha_w)\right)^2}{[\Gamma(1-\,\alpha_w) - (\ln 2)^{-\,\alpha_w}]^2} \cong 70.22. \tag{30}$$

Using FindRoot function, then

$$FindRoot\left[\frac{Gamma[1-2\,\alpha_w] - (Gamma[1-\,\alpha_w])^2}{(Gamma[1-\,\alpha_w] - (\mathrm{Log}[2])^{-\,\alpha_w})^2} == 70.22, \{\,\alpha_w, -0.5\}\right]$$

$$FindRoot\left[\frac{Gamma[1-2\,\alpha_w] - (Gamma[1-\,\alpha_w])^2}{(Gamma[1-\,\alpha_w] - (\mathrm{Log}[2])^{-\,\alpha_w})^2} == 70.22, \{\,\alpha_w, -0.8\}\right]$$

$$FindRoot\left[\frac{Gamma[1-2\,\alpha_w] - (Gamma[1-\,\alpha_w])^2}{(Gamma[1-\,\alpha_w] - (\mathrm{Log}[2])^{-\,\alpha_w})^2} == 70.22, \{\,\alpha_w, -1\}\right]$$

$$FindRoot\left[\frac{Gamma[1-2\,\alpha_w] - (Gamma[1-\,\alpha_w])^2}{(Gamma[1-\,\alpha_w] - (\mathrm{Log}[2])^{-\,\alpha_w})^2} == 70.22, \{\,\alpha_w, -1.7\}\right]$$

Thus, we obtain the values of the shape parameter (α_w)

$\alpha 1_w = -0.5068,\ \alpha 2_w = 0.4765,\ \alpha 3_w = -0.0844,\ \alpha 4_w = -3.9986.$

Hence, the corresponding values of the location and scale parameters (λ_w and θ_w), respectively are:

If $\alpha 1_w = -0.5068$, then $\lambda 1_w = 9.39$ and $\theta 1_w = -2.12$,

If $\alpha 2_w = 0.4765$, then $\lambda 2_w = 8.59$ and $\theta 2_w = 0.22$,

If $\alpha 3_w = -0.0844$, then $\lambda 3_w = 8.07$ and $\theta 3_w = 1.69$, If $\alpha 4_w = -3.9986$, then $\lambda 4_w = 8.69$ and $\theta 4_w = -0.04$.

In the GEV, $\boldsymbol{F}(\boldsymbol{x})$ is not a cdf if

$\alpha 1_w = -0.5068$, $\lambda 1_w = 9.39$, $\theta 1_w = -2.12$, also if $\alpha 4_w = -3.9986$, $\lambda 4_w = 8.69$, $\theta 4_w = -0.04$

While, $F(x)$ is a cdf if

$\alpha 2_w = 0.4765$, $\lambda 2_w = 8.59$, $\theta 2_w = 0.22$, also if $\alpha 3_w = -0.0844$, $\lambda 3_w = 8.07$, $\theta 3_w = 1.69$

Similarly, from Eqs. (12), (13) and (14) the mean, median, and variance are given by

$$Mean = \alpha_w + \lambda_w \Gamma\left(1 + \frac{1}{\beta_w}\right) = 8.9146 \tag{31}$$

$$Median = \alpha_w + \lambda_w (\ln 2)^{\frac{1}{\beta_w}} = 8.68 \tag{32}$$

$$Variance = \lambda_w^2 \left(\Gamma\left(1 + \frac{2}{\beta_w}\right) - \Gamma^2\left(1 + \frac{1}{\beta_w}\right)\right) = 3.8637. \tag{33}$$

Hence, from Eqs. (31) and (32)

$$\lambda_w \left[\Gamma\left(1 + \frac{1}{\beta_w}\right) - (\ln 2)^{\frac{1}{\beta_w}}\right] = 0.2346$$

$$\lambda_w = \frac{0.2346}{\Gamma\left(1 + \frac{1}{\beta_w}\right) - (\ln 2)^{\frac{1}{\beta_w}}}$$

$$\lambda_w^2 \cong \frac{0.0550}{\left[\Gamma\left(1 + \frac{1}{\beta_w}\right) - (\ln 2)^{\frac{1}{\beta_w}}\right]^2}. \tag{34}$$

Substituting Eq. (34) in Eq. (33), then

$$\frac{\Gamma\left(1 + \frac{2}{\beta_w}\right) - \Gamma^2\left(1 + \frac{1}{\beta_w}\right)}{\left[\Gamma\left(1 + \frac{1}{\beta_w}\right) - (\ln 2)^{\frac{1}{\beta_w}}\right]^2} \cong 70.22. \tag{35}$$

Using FindRoot function, then

$$FindRoot\left[\frac{Gamma\left[1 + \frac{2}{\beta_w}\right] - \left(Gamma\left[1 + \frac{1}{\beta_w}\right]\right)^2}{\left(Gamma\left[1 + \frac{1}{\beta_w}\right] - (\text{Log}[2])^{\frac{1}{\beta_w}}\right)^2} == 70.22, \{\lambda_w, -0.3\}\right]$$

$$FindRoot\left[\frac{Gamma\left[1+\frac{2}{\beta_w}\right]-\left(Gamma\left[1+\frac{1}{\beta_w}\right]\right)^2}{\left(Gamma\left[1+\frac{1}{\beta_w}\right]-(\mathrm{Log}[2])^{\frac{1}{\beta_w}}\right)^2}==70.22,\{\lambda_w,0.5\}\right]$$

$$FindRoot\left[\frac{Gamma\left[1+\frac{2}{\beta_w}\right]-\left(Gamma\left[1+\frac{1}{\beta_w}\right]\right)^2}{\left(Gamma\left[1+\frac{1}{\beta_w}\right]-(\mathrm{Log}[2])^{\frac{1}{\beta_w}}\right)^2}==70.22,\{\lambda_w,1\}\right]$$

Hence, we obtain the values of the scale parameter (λ_w)

$\lambda 1_w = 11.8459$, $\lambda 2_w = -2.0989$, $\lambda 3_w = 1.9733$.

We obtain the values of the shape parameter ($\boldsymbol{\beta_w}$) by using FindRoot function

$$FindRoot\left[Gamma\left[1+\frac{1}{\beta_w}\right]-(\mathrm{Log}[2])^{\frac{1}{\beta_w}}==0.0198,\{\lambda_w,-0.3\}\right]$$

$$FindRoot\left[Gamma\left[1+\frac{1}{\beta_w}\right]-(\mathrm{Log}[2])^{\frac{1}{\beta_w}}==-0.1118,\{\lambda_w,0.5\}\right]$$

$$FindRoot\left[Gamma\left[1+\frac{1}{\beta_w}\right]-(\mathrm{Log}[2])^{\frac{1}{\beta_w}}==0.1189,\{\lambda_w,1\}\right]$$

Then, the corresponding values of the shape and location parameters (β_w and α_w), respectively will be:

If $\lambda 1_w = 11.8459$, then $\beta 1_w = -0.4081$ and $\alpha 1_w = -20.4125$,

If $\lambda 2_w = -2.0989$, then $\beta 2_w = 7.294$ and $\alpha 2_w = 10.8822$,

If $\lambda 3_w = 1.9733$, then $\beta 3_w = 1.503$ and $\alpha 3_w = 7.1336$,

In the W-3P, $\boldsymbol{F(x)}$ is not a cdf if

$\lambda 1_w = 11.8459, \beta 1_w = -0.4081$, $\alpha 1_w = -20.4125$, also if $\lambda 2_w = -2.0989$, $\beta 2_w = 7.294$, $\alpha 2_w = 10.8822$

While, $F(x)$ is a cdf if

$\lambda 3_w = 1.9733, \beta 3_w = 1.503, \alpha 3_w = 7.1336$

Consequently, the actual dataset of the MWS simply belongs to the interval [3.59, 16.54]. Table (1) reveals how each of the mean, median, variance, standard deviations, and standard error of the GEV and W-3P models are computed for the MWS. Furthermore, the GEVD is compared with W-3P distribution. The results of the method used in this study for the GEVD, 8.9144, 8.6799, 3.8606, 1.9649 and 0.103991, got from ($\boldsymbol{\alpha 3_w}$ = -0.0844, $\boldsymbol{\lambda 3_w}$ = 8.07, $\boldsymbol{\theta 3_w}$ = 1.69), are closer to the actual values of the mean, median, variance, standard deviations, and standard error. For this reason, the values of ($\boldsymbol{\alpha 2_w}$ = 0.4765,

$\boldsymbol{\lambda 2_w}$ = 8.59, $\boldsymbol{\theta 2_w}$ = 0.22) and ($\boldsymbol{\beta 3_w}$ = 1.503, $\boldsymbol{\lambda 3_w}$ = 1.9733, $\boldsymbol{\alpha 3_w}$ = 7.1336), for the GEV and W-3P distributions respectively are excluded, whereas the values of ($\boldsymbol{\alpha 3_w}$ = -0.0844, $\boldsymbol{\lambda 3_w}$ = 8.07, $\boldsymbol{\theta 3_w}$ = 1.69) are accepted due to it is more accurate. It is clear from Table (1) that GEVD model is the best for modeling MWS data where it has the closest statistics with the actual values. In addition, the results indicate that the estimates of the used method are nearer with the actual values than MLE method, confirming the efficiency of moment generating function in estimating parameters. Finally, GEVD model is the most appropriate model for fitting MWS data.

Table 1: Comparison of the mean, median, variance, standard deviations, and standard error of the GEV and W-3P models for MWS data.

Statistics / Model (Method)	Mean	Median	Variance	St. Dev.	St. Error
Compute from the actual dataset	8.9146	8.68	3.8637	1.9656	0.104032
GEV (Compute from the used method in this study), where $\alpha 2_w$ = 0.4765, $\lambda 2_w$ = 8.59, $\theta 2_w$= 0.22	8.9107	8.6781	3.8003	1.9494	0.103175
GEV (Compute from the used method in this study), where $\alpha 3_w$= -0.0844, $\lambda 3_w$ = 8.07, $\theta 3_w$ = 1.69	8.9144	8.6799	3.8606	1.9649	0.103991
GEV (Compute from the MLE method), where: $\alpha 4_w$= -0.13353, $\lambda 4_w$ = 8.1045, $\theta 4_w$ = 1.7639	8.9146	8.73543	3.82831	1.9566	0.103555
W-3P (Compute from the used method in this study), where $\beta 3_w$=1.503, $\lambda 3_w$=1.9733, $\alpha 3_w$= 7.1336	8.9146	8.67988	1.45683	1.20699	0.063881
W-3P (Compute from the MLE method), where: $\beta 3_w$ = 3.1263, $\lambda 3_w$ = 6.4411, $\alpha 3_w$ = 3.1398	8.9024	8.86836	4.07373	2.01835	0.106822

And accordingly, the cdf in Eq. (3) is:

$$\boldsymbol{F_{GEV}(x;\ -0.0844, 1.69, 8.07) = e^{-\left(1-(0.0844)\left(\frac{x-8.07}{1.69}\right)\right)^{\frac{1}{0.0844}}}}. \qquad (\boldsymbol{36})$$

Correspondingly, the pdf of the GEVD in Eq. (1) is defined as:

$$\begin{bmatrix} f_{GEV}(x;\ -0.0844, 1.69, 8.07) = \frac{1}{1.69}\ g(x)^{0.9156}\ e^{-g(x)}, \\ where,\ g(x) = \left(1 - \frac{0.0844(x - 8.07)}{1.69}\right)^{\frac{1}{0.0844}} \end{bmatrix}. \quad (37)$$

Table (2) provides the GEVD comparison with W-3P model. Also, the Kolmogorov-Smirnov (K-S) and Anderson-Darling (A*) tests are computed. The K-S test is defined as follows

H_{01}: the dataset of the MWS follows the GEVD.

H_{02}: the dataset of the MWS follows the W-3P.

H_{a1}: the dataset does not follow the GEVD.

H_{a2}: the dataset does not follow the W-3P.

Table 2: Results of A*and K-S test of the GEVD and W-3P for MWS using (Significance Level ($\alpha = 0.05$) and Critical Values ($D_n^{\alpha} = 0.0720$)).

Models	A*	K-S (Moment Function)	K-S (MLE)
GEVD	0.69498	0.0359 (Accept)	0.04071 (Accept)
W-3P	2.1735	0.139 (Reject)	0.06534 (Accept)

It is clear from Table (2) that GEVD model is the best for fitting MWS data where it has the least K-S and A* values. In this regard, H_{01} is accepted, and the actual dataset follows the GEVD. While, the K-S test statistic (D_n) of the moment generating function of the GEVD is equal to 0.0359 is less than the D_n of MLE, then it is more acceptable than MLE method. In addition, H_{02} is accepted only for MLE method, but using the moment function with W-3P distribution isn't suitable for modeling MWS data.

Similarly, the GEVD is compared with other competitive models as, three-parameter Gamma (G-3P), generalized Gamma (GG), Kumaraswamy (K), three-parameter Erlang (E-3P), and Fréchet (F) distributions [22, 23, 24, 25, 26]. The mean, variance, estimations of the parameters, A* and K-S are displayed in Tables (3), (4) and (5). The statistics in Table (3) indicate that the GEVD are nearer with the actual values than other rival distributions. According to Table (5) the GEVD is the most effective model for fitting MWS data where it has the lowest values of A* and K-S.

Table 3: Comparing the mean and variance for GEVD and other rival distributions for MWS.

Statistics / Models	Mean	Variance
Compute from the actual dataset	8.9146	3.8637
GEVD	8.9144	3.8606
G-3P	8.9145	3.8437
GG	8.8814	181.806
K	0.2632	0.0083
E-3P	0.1538	0.0099
F	8.9784	6.9133

Table 4: Estimating the parameters of GEVD and other models for MWS data.

Models	Parameters
GEVD	α_w = -0.13353, θ_w = 1.7639, λ_w = 8.1045
G-3P	α_w = 36.79, β_w = 0.32323, γ_w = -2.9771
GG	k_w = 0.99823, α_w = 20.399, β_w = 0.43463
K	a_w = 3.143, b_w = 46.174
E-3P	m_w = 37, β_w = 0.32323, γ_w = -2.9771
F	α_w = 5.2845, β_w = 7.7914

Table 5: Results of A*and K-S of the GEVD and other competing distributions for MWS data.

Models	A*	K-S
GEVD	0.69498	0.0359
G-3P	0.95388	0.04608
GG	0.82598	0.04326
K	2.2148	0.06595
E-3P	1.272	0.05867
F	7.7458	0.09815

The Kruskal-Wallis test is used to validate the efficiency of the GEVD for fitting MWS, then: P-Value = 0.942 > 0.05, the GEVD is accepted. The GEV pdf and cdf are displayed in Fig. (2) and Fig. (3), respectively. Fig. (4) and Fig. (5) shows the P-P plot and the Q-Q plot of the GEVD for MWS data in Port Said, Egypt. It is clear from Fig. (2), Fig. (3), Fig. (4) and Fig. (5) that GEVD is suitable for modeling MWS data in Port Said, Egypt.

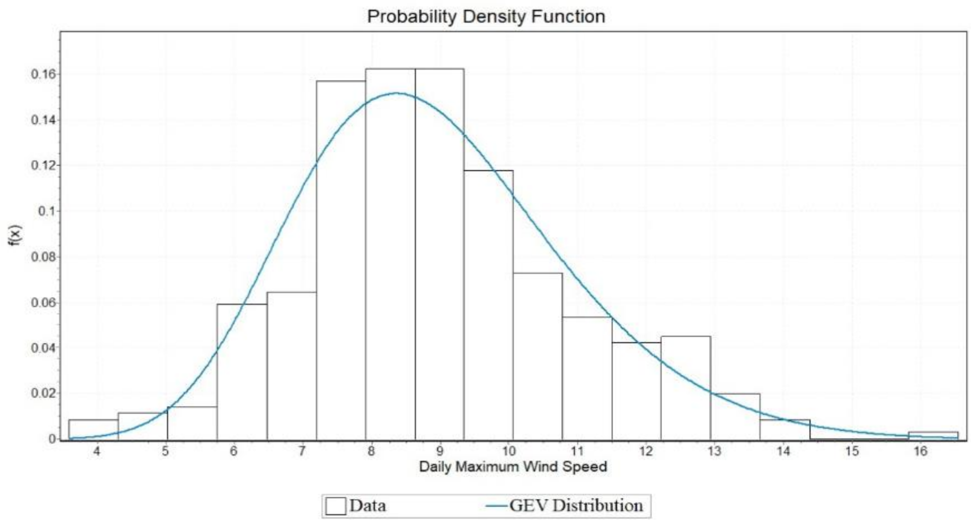

Fig. 2: Illustrates the pdf of GEVD of the MWS.

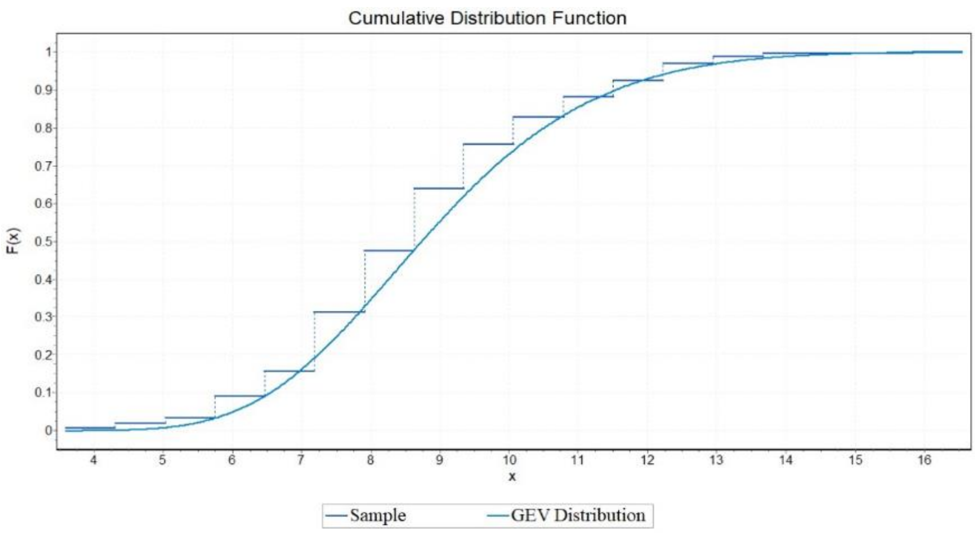

Fig. 3: Illustrates the cdf of GEVD of the MWS.

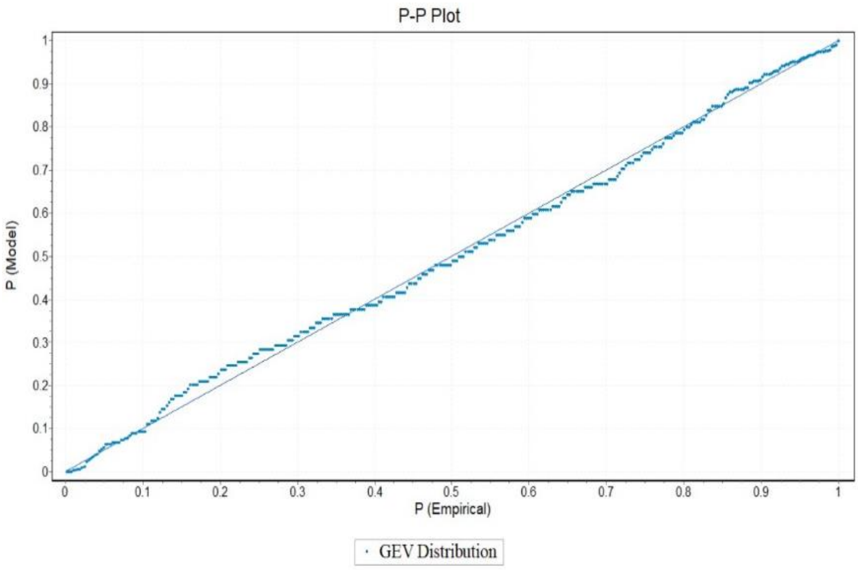

Fig. 4: Illustrates the P-P plot of the GEVD of the MWS data.

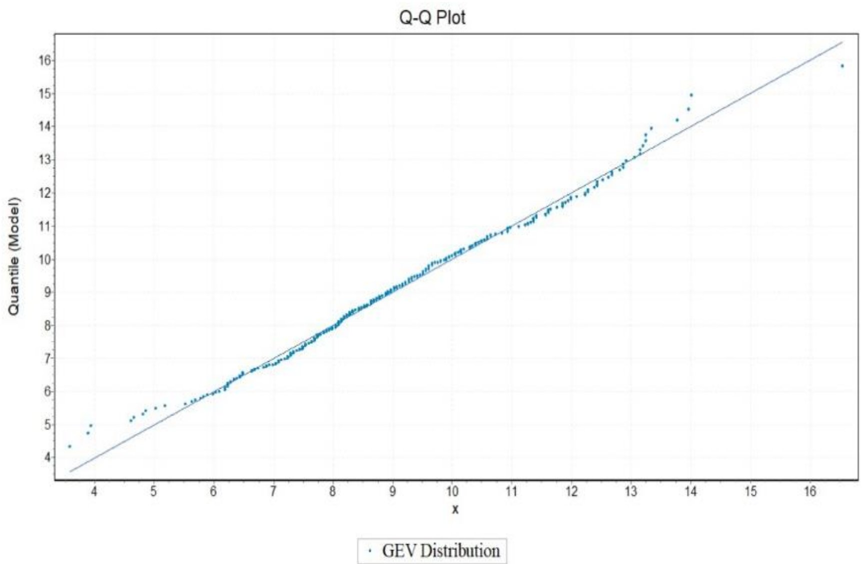

Fig. 5: Illustrates the Q-Q plot of the GEVD of the MWS data.

On the other hand, the comparison of the percentiles $\boldsymbol{P}(\boldsymbol{x})$ and quartiles $\boldsymbol{Q}(\boldsymbol{x})$ of the actual dataset of MWS with the theoretical percentiles $\boldsymbol{P}^*(\boldsymbol{x})$ and theoretical quartiles $\boldsymbol{Q}^*(\boldsymbol{x})$ for GEVD are shown in Tables (6) and (7), respectively.

Table 6: Comparing the percentiles for MWS.

$P(x)$ Actual Dataset	$P^*(x)$ GEVD	$P(x)$ Actual Dataset	$P^*(x)$ GEVD
6.49	6.6097	9.10	9.1736
7.14	6.9581	9.33	9.4438
7.44	7.2494	9.61	9.7386
7.69	7.5103	9.91	10.0686
7.87	7.7538	10.32	10.4510
8.09	7.9877	10.92	10.9168
8.19	8.2172	11.56	11.5337
8.43	8.4467	12.28	12.5098
8.63	8.6799	12.91	13.6882
8.88	8.9208	--	--

Table 7: Comparing the quartiles for MWS.

$Q(x)$ Actual Dataset	$Q^*(x)$ GEVD
7.69	7.5103
8.68	8.6799
10.05	10.0686
16.54	14.5125

As shown in Fig. (6), the $\boldsymbol{F(x)}$ of the actual dataset of the MWS can be compared with the theoretical cdf $\boldsymbol{F^*(x)}$, where $\boldsymbol{x}$ belongs to the interval [6.69, 13.34].

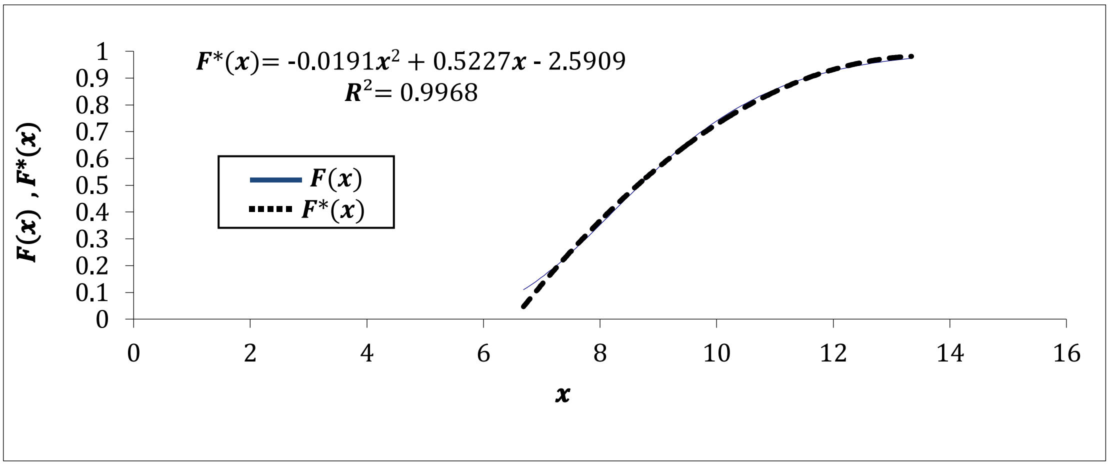

Fig. 6: $F(x)$ for actual dataset with theoretical function $F^*(x)$ of GEVD.

4. CONCLUSION

This study proposed an accurate method for estimating the parameters of the generalized extreme value distribution (GEVD) that describes the actual dataset for the MWS during the year 2015 in Port Said city using its moments-generating function. Also, it attempted to simplify the parameter's equations of the MWS distribution. Moreover, it managed to find the best estimating parameters suitable for the dataset. In comparison to W-3P and other well-known distributions, the GEVD model offers a best fitting to MWS data. In addition, it will help the researchers uncover the critical areas of estimation methods for the different distributions by reducing three parameters to one parameter. It is also important for researchers in ecology insofar as it helps arrive at reliable predictions of the MWS in different areas of the globe.

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