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Shock waves in ionosphere

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ABSTRACT

Fully nonlinear shock waves (SW) structure of ion-acoustic waves are investigated in a three-component cold plasma consisting of two positive ions fluids and non-thermal electron distribution. The physical parameters in the system, such as ion density ratios, furthermore, they play an important role in the profile of the small amplitude ion-acoustic SW. Using m-KdV, the basic equations are reduced to one evolution equation. The latter has been analyzed and solved numerically to obtain an arbitrary amplitude shock wave profile as well as the possible regions for the existing waves. We present a negative potential, which corresponds to a compressive wave profile. The findings of this investigation are used to interpret the electrostatic waves that may be observed in the Mars ionosphere. If we observe this wave in Mars's ionosphere, we will be able to explain how the gas has been lost from the ionosphere if we get to the physical meaning of it.

Key Words:

ion-acoustic waves; Mars ionosphere; Small amplitude Shock waves.

1. INTRODUCTION

It is essential for the entire area of space physics to understand the nonlinear events that occur in the planet's ionosphere. This nonlinearity is dependent on both the characteristics of the planetary obstruction and the features of all the physical plasma parameters. Many researchers have used fluid equations and magnetohydrodynamic equations to study the plasma phenomena in the planet's ionosphere. Understanding various astrophysical, space events, and commercial physical applications requires investigating a plasma containing two positive ions [1,2].

The formation of shock waves has been a hot topic in recent years [3,4] due to its importance in charged particle acceleration in cosmic plasmas and plasma thrusts for plasma space features. Therefore, investigating shock waves in the Cold ions could modify the plasma modes. For example, Witt and

Hudson [5] investigated the shock solutions produced by the two various plasma models: Cold ions, hot Boltzmann ions, cool Boltzmann electrons, and cold streaming electrons. Cold ions and two Boltzmann populations of electrons. Schott has studied plasma consisting of cold ions, with Maxwellian distribution [6]. One of the main study topics in space physics is the coupling of the tenuous and heated magnetospheric plasma to the dense and cold ionospheric plasma. The topside ionosphere is thought to transition at a strong double layer confined within 10 Debye lengths [7].

A double layer and a shock wave have different causes and effects. A double layer refers to a slender layer of plasma in which the electric field is responsible for accelerating both electrons and ions. Double layers can arise spontaneously in plasma and are known to have unique properties, such as their ability to maintain their structure and to self-organize. Understanding the behavior and properties of double layers is of great importance in various fields, including plasma physics, space science, and fusion research [8]. It is noteworthy that a shock wave is a sudden shift in the pressure, density, and temperature of a medium. This phenomenon is generally caused by a disturbance that propagates at a velocity higher than the speed of sound in the medium. It is a fascinating subject that warrants further examination [9].

The ionosphere is an intriguing and dynamic layer of planet's atmosphere that plays a crucial role in our understanding of the interactions between the Planet and space. This region, situated between the mesosphere and exosphere, is characterized by its ionized particles, plasma waves, and varying altitudes. Characterized by the presence of ionized particles, the ionosphere is an electrically charged region. This ionization occurs as interaction of neutral atoms and molecules in the upper atmosphere, causing them to lose or gain electrons. As a result, free electrons and two positive ions are formed. In this paper, we study fully nonlinear supersonic shock waves that consist of two positive cold ions $(O^+,\ H^+)$ with nonthermal electron that may be found in Planet ionosphere.

The skeleton of the paper is as follows: in Section 2, the basic set of fluid equations describing the system is presented. Then the mKdV equation is deduced. Section 3 has the mathematical solution and discussion. Section 4 contains conclusions of the paper.

2. Plasma Model

We consider three components collisionless, unmagnetized cold plasma in ionosphere environment having two positive ions and a nonthermal electron distribution The normalized continuity and momentum equations for H^+ions are describe as

$$\frac{\partial n_p}{\partial t} + \frac{\partial}{\partial x} n_p u_p = 0 \,, \tag{1}$$

$$\left(\frac{\partial u_p}{\partial t} + u_p \frac{\partial u_p}{\partial x}\right) + \frac{1}{\mu_p}\frac{\partial \phi}{\partial x} = 0 \,. \tag{2}$$

The basic fluids equations for the O^+ ion are

$$\frac{\partial n_n}{\partial t} + \frac{\partial}{\partial x} n_n u_n = 0 \,, \tag{3}$$

$$\left(\frac{\partial u_n}{\partial t} + u_n \frac{\partial u_n}{\partial x}\right) + \frac{\partial \phi}{\partial x} = 0 \,. \tag{4}$$

The electrons have non-thermal distributions

$$n_e = (1 - \beta\phi + \beta\phi^2)\exp(\phi) \,, \tag{5}$$

where $\gamma > 0$, and $\beta = 4\gamma/(1+3\gamma)$. For $\beta > 4/7$, a physically appropriate distribution function is acceptable . The system of Eqs. (1)–(5) is closed by the Poisson equation

$$\frac{\partial^2\phi}{\partial x^2} = \alpha_e n_e - \alpha_p n_p - n_n \,. \tag{6}$$

Here, n_p is Hydrogen ion number density, n_n is the Oxygen ion number density, n_e is electron number density, ϕ is the electrostatic potential, K_B is the Boltzmann constant, where $\mu_p = m_p/m_n$, $\alpha_p = n_{po}/n_{no}$, $\alpha_e = n_{eo}/n_{no}$, are the ratios of unperturbed charges densities-to-positive ion density. The Debye length $\lambda_{De} = \left(\frac{K_B T_e}{4\pi e^2 n_n}\right)^{1/2}$, the inverse of the ion plasma frequency $\omega_{pp}^{-1} = \left(\frac{m_n}{4\pi e^2 n_n}\right)^{1/2}$, and $C_{so} = \left(\frac{K_B T_e}{m_n}\right)^{1/2}$ is the ion-acoustic speed. To study the SWs with two positive ions fluids in the planet ionosphere with electrons have non-thermal distributions, the reductive perturbation method is used. Thus, the following space-time variables are introduced

$$\xi = \varepsilon(x - \lambda t), \quad \text{and } \tau = \varepsilon^3 \lambda t, \tag{7}$$

where λ is the phase velocity of the ion acoustic wave and ε is the magnitude of the perturbation. In addition to, all physical quantities are asymptotically stretched as power series in ϵ about their equilibrium values. The physical quantities appearing in Eqs. (1)–(6)

$$\begin{bmatrix} n_p \\ u_p \\ n_n \\ u_n \\ n_e \\ \phi \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} + \varepsilon \begin{bmatrix} n_{p1} \\ u_{p1} \\ n_{n1} \\ u_{n1} \\ n_{e1} \\ \phi_1 \end{bmatrix} + \varepsilon^2 \begin{bmatrix} n_{p2} \\ u_{p2} \\ n_{n2} \\ u_{n2} \\ n_{e2} \\ \phi_2 \end{bmatrix} + \varepsilon^3 \begin{bmatrix} n_{p3} \\ u_{p3} \\ n_{n3} \\ u_{n3} \\ n_{e3} \\ \phi_3 \end{bmatrix} + \ldots, \tag{8}$$

The charge-neutrality condition is maintained through the relation

$$\alpha_e - \alpha_p - 1 = 0. \tag{9}$$

Using Eqs. (8) and (9) into Eqs. (1)–(6) and collecting the highest-order in ϵ, we get

$$n_p^{(2)} = \frac{3}{2\lambda^3\mu_p^2}\phi^{(1)^2} + \frac{1}{\lambda^2\mu_p}\phi^{(2)}, \tag{10}$$

$$u_p^{(2)} = \frac{1}{2\lambda^2\mu_p^2}\phi^{(1)^2} + \frac{1}{\lambda\mu_p}\phi^{(2)}, \tag{11}$$

$$n_n^{(2)} = \frac{3}{2\lambda^3}\phi^{(1)^2} + \frac{1}{\lambda^2}\phi^{(2)}, \tag{12}$$

$$u_n^{(2)} = \frac{1}{2\lambda^2}\phi^{(1)^2} + \frac{1}{\lambda}\phi^{(2)}, \tag{13}$$

and the Poisson equation gives the dispersion relation

$$\frac{\alpha_p}{2(\lambda^2\mu_p)} + \frac{1}{2\lambda^2} + \frac{\alpha_p+1}{2} = 0. \tag{14}$$

The next-height order of the perturbation gives the m-KdV equation

$$\frac{\partial\phi^{(1)}}{\partial t} + \left(AB\phi^{(1)} + AC\phi^{(1)^2}\right)\frac{\partial\phi^{(1)}}{\partial\xi} + \frac{1}{2}A\frac{\partial^3\phi^{(1)}}{\partial\xi^3} = 0, \tag{15}$$

Eq. (15)'s steady-state solution can be obtained by applying the transformation.

$$\eta = \xi - U\tau, \tag{16}$$

where U refers to a constant velocity. Boundary conditions are used $\phi \to 0$ and $d\phi/d\eta \to 0$ at $\eta \to \pm\infty$ to get

$$\frac{1}{2}\left(\frac{d\phi}{d\eta}\right)^2 + V(\phi) = 0. \tag{17}$$

Where the Sagdeev pseudo potential is

$$V(\phi) = \frac{-U}{A}\phi^{(1)^2} + \frac{1}{3}B\phi^{(1)^3} + \frac{1}{6}C\phi^{(1)^4}, \tag{18}$$

where

$$A = \left(\frac{\alpha_p}{\lambda^2\mu_p} + \frac{1}{\lambda^2}\right)^{-1}, \tag{19}$$

$$B = \frac{1}{2}\left(\frac{3\alpha_p}{\lambda^4\mu_p^2} + \frac{3}{\lambda^4} + (3\beta+1)\alpha_e\right), \tag{20}$$

$$C = \frac{3}{4}\left(\frac{5\alpha_p}{\lambda^6\mu_p^3} + \frac{5}{\lambda^5} + (8\beta+1)\alpha_e\right). \tag{21}$$

3. Mathematical Solution and Discussion

The Sagdeev potential should be negative for the shock waves solution between $\phi = 0$ and ϕ_m, where ϕ_m is a maximum potential value. We use the plasma data from Ref. [11]: $\alpha_p = 0.75$, $\beta = 0.38$. The Sagdeev pseudo potential needs to fulfill the following conditions to allow the formation of SWs:

$$V(\phi) = 0 \quad at \quad \phi = 0 \quad and \quad \phi = \phi_m, \tag{22}$$

$$V'(\phi) = 0 \quad at \quad \phi = 0 \quad and \quad \phi = \phi_m, \tag{23}$$

$$V''(\phi) < 0 \quad at \quad \phi = 0 \quad and \quad \phi = \phi_m. \tag{24}$$

By using the boundary condition (22) and (23) in Eq. (18), we get

$$V = \frac{-AC}{6}\phi_m^2 \quad and \quad \phi_m = \frac{-B}{C}. \tag{25}$$

If we substitute V and ϕ_m from equation (25) into equation (18), we have

$$V(\phi) = \frac{C}{6}\phi_m^2(\phi_m - \phi). \tag{26}$$

The shock waves solution of Eq. (17), with Eq. (26), is

$$\phi = \frac{\phi_m}{2}\left[1 - \tanh\left(\sqrt{\frac{-C}{8}}\ \phi_m\ \eta\right)\right]. \tag{27}$$

It is noticeable that $c < 0$ is necessary for the presence of a shock wave. Equation (25) additionally shows that the type of the SWs depends on the sign of B; for example, a compressive shock waves exists for $B > 0$ whereas a rarefactive SWs would exist for $Q < 0$. The shock wave's width is determined by

$$W = \frac{2\sqrt{-8/C}}{|\phi_m|}. \tag{28}$$

Figure (1) represent the profile of shock wave potential ϕ is depicted against normalized spatial coordinate η for different value of α_p and β as Fig. 1(a) show the increase in density ratio α_p leads to increasing in amplitude and wave width which mean the density ratio is increasing the non-linearity of system the same effect in Fig. 1(b) increasing the non-thermal electron density β incease the non-linearity of system. The uni-polar electric field is present in Fig. (2) against normalized spatial coordinate η in different values of density ratio α_p and non-thermal electron density β. In Fig. 2(a) it can be seen the increase in density ratio α_p associated by increasing in the amplitude of the electric field the same effect in Fig. for β physically the increase in α_p and β increase the non-linearity Which in turn increase the electric field associate with wave. In Fig. (3) we apply fast Fourier transformation to obtain the normalized electric field as we see in Figs. 3(a) and 3(b) the associated electric field to shock waves is about $40\ mV/m - 80\ mV/m$ and has a time duration of about $4\ ms$. From this result, we found the relaxation time was greater than the phenomenon time, so we used a non-thermal distribution. Figure (4) The corresponding fast Fourier transform (FFT) power spectra of the electric fields frequency range of 0.1 to 2 kHz. We predict that type of non-linear wave can be found in Mars's Ionosphere Understanding the physics behind that we can be familiar with how the gas is lost from the ionosphere.

4. Summary

In a plasma containing two positive ions, we examined the properties of shock waves. The Sagdeev potential has been obtained using the reductive modulation method to obtain a solution for SWs. The SWs behavior has been investigated with regard to the effects of a concentration of positively charged ions and non-thermal electron density. The results show that the density ratio has an important influence on the amplitude and width of SWs, along with nonthermal electron density. The system supports rarefactive SWs. In conclusion, we stress that the results of the present investigation are important for understanding the properties of shock waves in ionosphere plasma that consist of two positive cold ions with non-thermal electron distribution and it can be used to recognize a possible nonlinear wave at Mars' ionosphere.

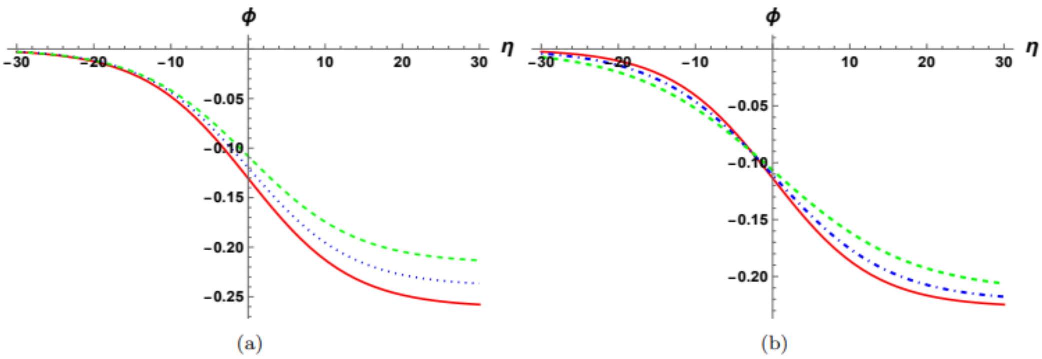


Figure 1: (a) Is a shock wave profile wave potential ϕ is depicted against normalized spatial coordinate η for α_p = 0.7 (dashed-green), α_p = 0.75 (dotted-blue), α_p = 0.8 (red).
(b) is a shock wave profile wave potential β for β = 0.28 (dashed-green), β = 0.38 (dotted-blue), β =0.48 (red).

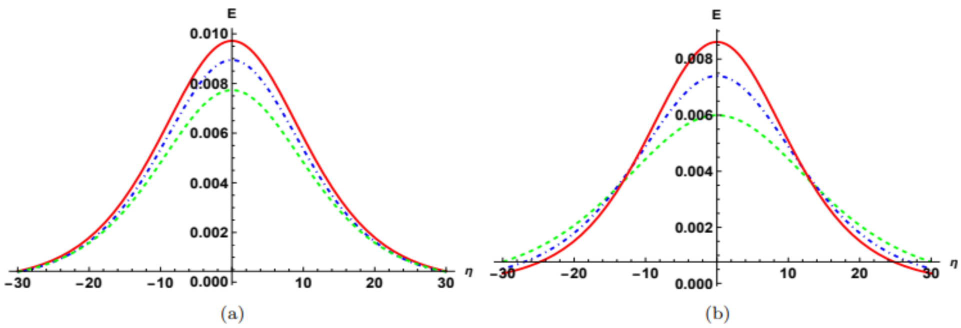
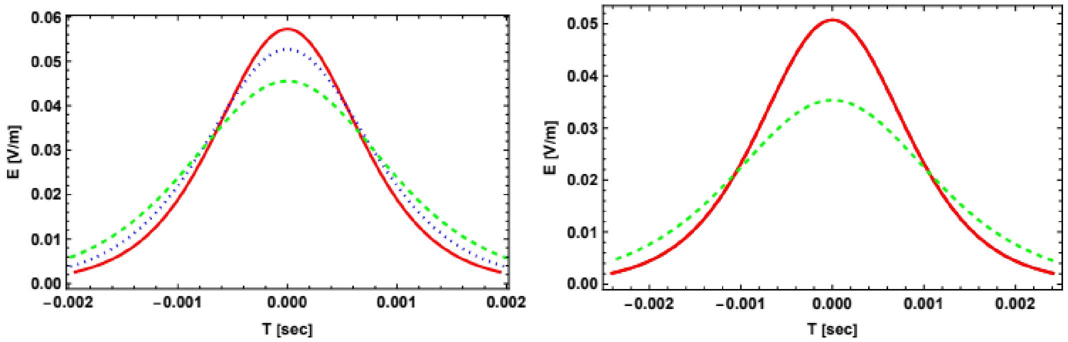
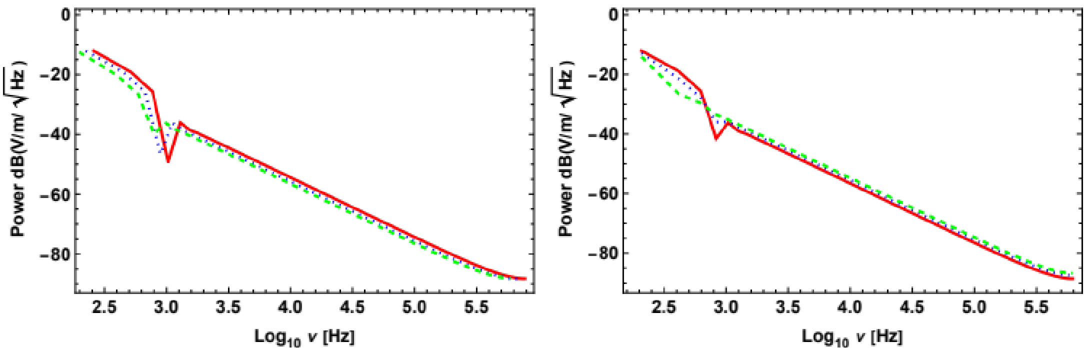


Figure 2: (a) Is the normalized electric field E is depicted against normalized spatial coordinate η for α_p = 0.7 (dashed-green), α_p = 0.75 (dotted-blue), α_p = 0.8 (red).
(b) is a shock wave profile wave potential β for β = 0.28 (dashed-green), β = 0.38 (dotted-blue), β = 0.48 (red).



(a) (b)

Figure 3: The associated uni-polar electric field. E is depicted against time duration for **(a)** $\alpha_p = 0.7$ (dashed-green), $\alpha_p = 0.75$ (dotted-blue), $\alpha_p = 0.8$ (red). **(b)** β for $\beta = 0.28$ (dashed-green), $\beta = 0.38$ (dotted-blue), $\beta = 0.48$ (red).



(a) (b)

Figure 4: The corresponding fast Fourier transform (FFT) power spectra of the electric fields. **(a)** for $\alpha_p = 0.7$ (dashed-green), $\alpha_p = 0.75$ (dotted-blue), $\alpha_p = 0.8$ (red).

(b) for $\beta = 0.28$ (dashed-green), $\beta = 0.38$ (dotted-blue), $\beta = 0.48$ (red).

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