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Thermal Evolution of f(R) Modified Gravity in the Frame of $S-M$ Holographic Dark Energy Model

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ABSTRACT

In this work we establish the main cosmological parameters namely the deceleration parameter q, the equation of state parameter w and the diagnostic omega Om for $f(R)$ modified gravity using Sharma-Mitall $(S-M)$ Holographic Dark Energy model $(SMHDE)$. Our model shows a type of stability is achieved by studying the behavior of the square speed of sound parameter which has a positive behavior over the given range. For the considered red shift z range, we study the evolution of energy conditions for our model and the growth of dark energy. The results fit the observations that the dark energy increase with time and the universe has an accelerated expansion epoch. Thermal analysis is considered through the quantum correction for power law and logarithmic entropy, consequently we study the validity of the generalized second law of thermodynamics the results show that the generalized second law of thermodynamics holds using power law corrected scenario.

Keywords
Sharma Mitall holographic Dark energy and $f(R)$ gravity model.
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1. INTRODUCTION

The accelerated expansion behaviour of the universe has attracted many scientises in the last 20 years. This behaviour is firstly observed in 1998 as a result a deep understanding for many universe phenomena takes place. Recently, physists assumed many models to understand universe features and trying to give some answer to some mystery questions [1, 2, 3, 4]. Dark energy (DE) is a one of the biggest challenging questions, it is considerd as the main candidate of accelerated expansion behavior of the universe. To understand its nature some models are proposed namely, scalar field models, dynamical dark energy models and modified gravity models. [5, 6, 7, 8, 9, 10, 11, 12].

The holographic dark energy model (HDE) assumed by M. li [13] as a promising model used to understand DE nature. This model based on the holographic principle [14], where the degrees of freedom of the system are represented by using the boundary area rather that its volume. For infrared (IR) cutoff equal to the size of universe the holographic energy density will be close to the dark energy density. The relation between holographic principle and other dark energy models are considerd in some pervious works [15, 16, 17, 18, 19, 20, 21].

Using a new entropy formula together with holographic principle, some new holographic dark energy models are established, like Tsallis HDE, Renyi HDE and Sharma Mitall Holographic Dark Energy (SMHDE) [22, 23, 24, 25].

In this paper, the behavior f(R) modified gravity model is studied within the context of $(S-M)$ Holographic Dark Energy, the structure of the paper is as follows. In section (II) we consider our model, in section (III) we study the diagnostic state finder parameter, the stability of the model considered using the square speed of sound and energy conditions and in section (IV) thermal evolution of the model are studied and in last section the conclusion is present.

2. THE MODEL

The general form action of $f(R)$ modified gravity is [26, 27, 28]:

$$S=\int dx^4\sqrt{-g}(f(R)+L_m), \tag{1}$$

where g is a coupling constant and L_m is the lagrangian of matter components. The corresponding field equation is given by:

$$R_{\Upsilon\vartheta}-(g_{\Upsilon\vartheta}\Box-\nabla_{\Upsilon}\nabla_{\vartheta})=T^{m}_{\Upsilon\vartheta}, \tag{2}$$

where T is the stress energy momentum tensor, $\Box$ and ∇ are d'Alembertian and laplacian operators and $R_{\Upsilon\vartheta}$ is the Ricci tensor. By action variation and by using the Friedman field equations $3H^2=\rho+\rho_m, -\dot{H}^2-3H^2=p+p_m$, one can write the energy density and pressure for $f(R)$ model as:

$$\rho=f(t)+H(t)f_{\mathrm{RR}}(t)\dot{R(t)}-f_R(t)\left(R(t)-6H(t)^2\right), \tag{3}$$

and

$$p=-f(t)+f_R(t)\left(-4\left(\dot{H(t)}+H(t)^2\right)-2H(t)^2+R(t)\right)-2f_{\mathrm{RRR}}(t)\dot{R(t)}+f_{\mathrm{RR}}(t)\left(-2\ddot{R(t)}-4\dot{R(t)}\right), \tag{4}$$

where, dot represents the first derivative with respect to cosmic time, H is the Hubble parameter, f_R, f_R, f_{RRR} are the first, second and third derivatives of $f(R)$ with respect to $R(t)$ respectively and $R(t)$ is given by:

$$R(t)=6\left(\dot{H(t)}+2H(t)^2\right). \tag{5}$$

In the current study, we use the form proposed by S. M. Carroll [29] for the late-time universe evolution:

$$f(R)=R(t)-\frac{H_o^4}{R(t)}, \tag{6}$$

Sharma and Mittal assumed parametric entropy in the form [24]:

$$S_{SM} = \frac{1}{\varpi}((1+\frac{\delta A}{4})^{\gamma} - 1), \tag{7}$$

where ϖ is a free parameter, $\gamma = \frac{\varpi}{\delta}$, δ is a real number and A is the horizon area. The entropy is related to UV-cutoff by using the holographic principle:

$$\Lambda^4 \leq \frac{S}{L^4}. \tag{8}$$

Following the HDE model, the dark energy is related to UV cutoff ($\rho \sim \Lambda^4$). For the infrared cutoff L equal to Hubble parameter (H) i.e $L = \dfrac{1}{H} = \sqrt{A/4\pi}$. Eqs. (7) and (8) can be written as [24]:

$$\rho = \frac{3c^2 H(t)^4 \left(\left(\frac{\pi\delta}{H(t)^2} + 1\right)^{\gamma} - 1\right)}{8\pi R}, \tag{9}$$

The fractional energy density is defined as:

$$\Omega = \frac{\rho}{3H(t)^2}, \tag{10}$$

where c^2 is a numerical parameter. Eq. (9) represents the Sharma Mitall Holographic Dark Energy (SMHDE). By choosing $R = \delta$ one can recover the ordinary HDE model. By combing Eqs. (3), (5), and (9) and after some algebraic calculations, we can establish the Hubble parameter for our model. In this work, we use the relation between the cosmic time t and red-shift z, namely $t = \frac{2}{((z+1)^2+1)H_o}$ and we consider the approximation $H_o = 1$ for our analysis. In Fig. 1 (a) we study the growth of the Hubble parameter H (b)the deceleration parameter $q = -\frac{\dot{H(t)}}{H(t)^2} - 1$. We observe that q decreases with the increase in z stays in negative that indicates accelerating expansion behavior in (c) the positive behavior of $\Omega' = \frac{\dot{\Omega}}{H}$ indicates the growth of dark energy with time.
Assuming DE dominate universe, we can write the non interacting equation of continuity as :

$$\dot{\rho} + 3H(\rho + p) = 0. \tag{11}$$

Therefore, one can write the equation of state parameter as:

$$\omega = \frac{p}{\rho} = -\frac{\dot{\rho} - 3H\rho}{3H\rho}. \tag{12}$$

In Fig. 1 (d) the evolution of equation of state parameter is studied showing a quintessence mode since $\omega > -1$.

3. DIAGNOSTIC OMEGA OM, STABILITY AND ENERGY CONDITIONS

Here, we are studying the geometrical state finder parameter called diagnostic-omega Om, this parameter is easier in treatment rather than the other geometrical parameters because it has no time derivative of Hubble parameter. The dyagnostic-Om is given by the formula [31]:

$$Om = \frac{\frac{H(z)^2}{H_o^2} - 1}{(z+1)^3 - 1}. \tag{13}$$

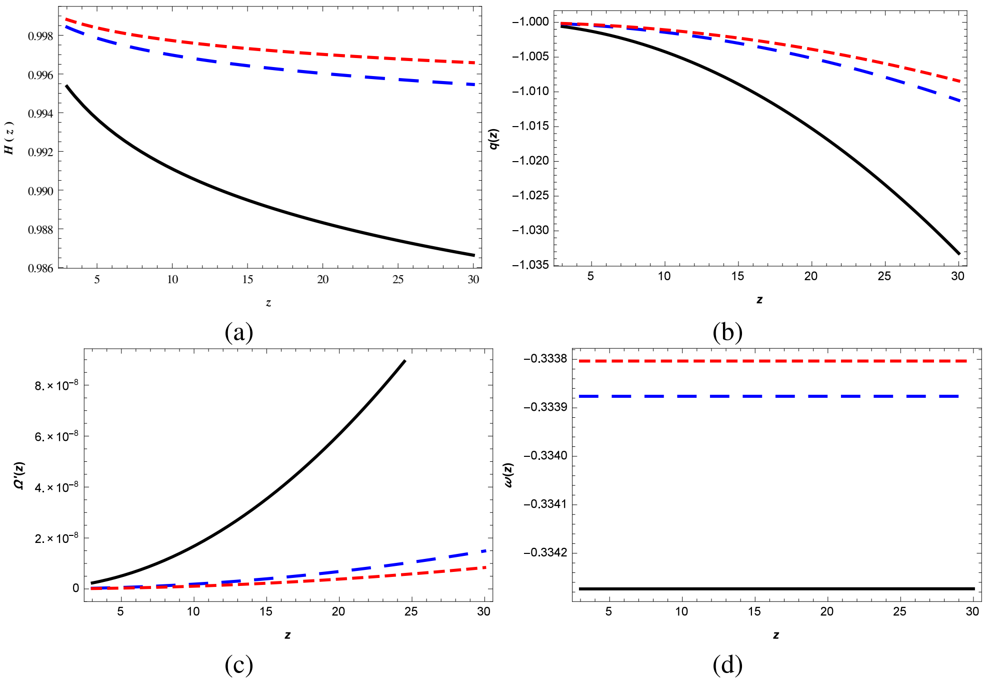


Figure 1: (a) The growth of H (b) the q (c) the $\Omega^{'}$ (d) the EoS as function of red-shift z. Red, blue, and black lines correspond to $\gamma = -200, -600$ and -800 for the $f(R)$ mode

If we consider a constant value for the equation of sate parameterω, one can write [31]:

$$Om = \Omega_{m0} + \frac{(1-\Omega_{m0})(1+z)^{3(1+\omega)} - 1}{(z+1)^3 - 1}. \tag{14}$$

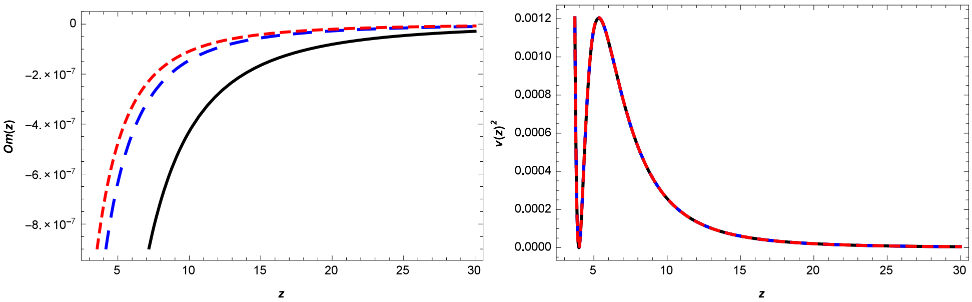
where Ω_{m0} is the present value of the matter density. We find $Om = \Omega_{m0}$ for ΛCDM, for$Om > \Omega_{m0}$ we have a quintessence like behavior $\omega >$ âĹŠ1 while for $Om < \Omega_{m0}$ we have phantom like behavior $\omega <$ âĹŠ1. That way this geometrical parameter is used to understand the difference between ΛCDM and the other dark energy models. In FIG. 2 (a) the evolution of Om as a function of red-shift z is considered. we notes that the Om increases with a decreasing rate over the considered range of red-shift z then tends to reach some constant value close to 0. The slope of this graph is used to understand the expansion nature of our universe. The positive slope leads to the phantom model while the negative slope indicates quintessence behavior.

The square speed of sound is used to test the stability of our model, it is denoted as v^2 and is given by [32]:

$$v^2 = \frac{\dot{P}}{\dot{\rho}} = \dot{\omega}\frac{\rho}{\dot{\rho}} + \omega. \tag{15}$$

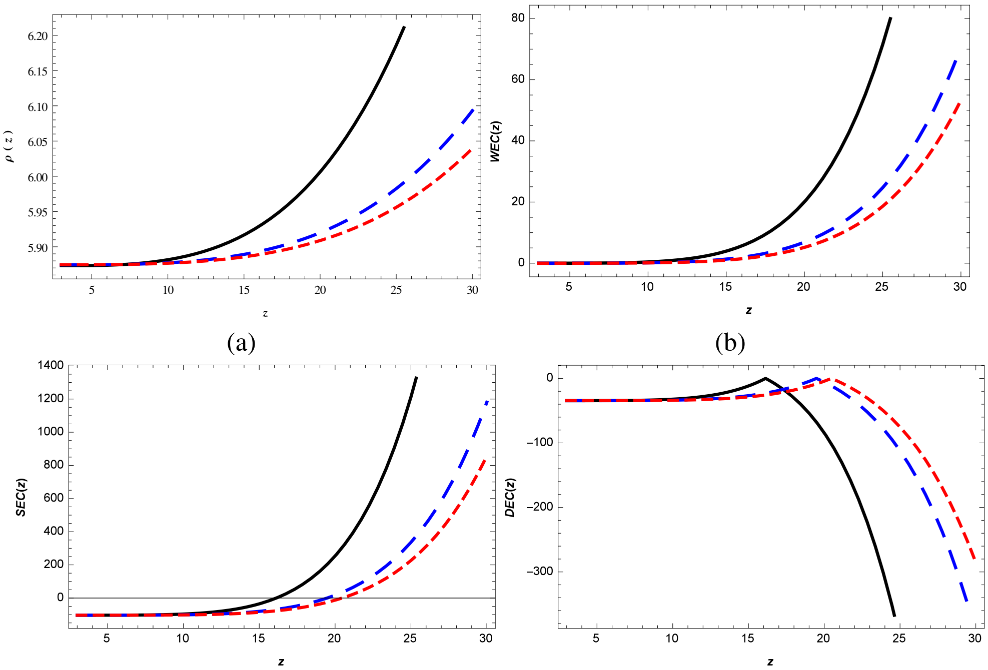
Now, we can check the stability of the model, simply for negative values of v^2 the model is unstable while for positive values the model is stable. In FIG. 2 (b) we study the growth of v^2 as a function of z. We notice that the behavior is independent on γ values and we observe v^2, shows a type of fluctuation between minimum and maximum values for $z < 6$ then shows a decreasing behavior with a decreasing rate but remains in positive level and therefore the model is stable.

Now, we consider the energy conditions where these conditions are important in understanding the charactristics of some important cosmological theorems about black holes [33]. The energy condi-



(a) (b)

Figure 2: (a) The growth of the Om (b) the v^2 against the red-shift z. Red, blue, and black lines correspond to $\gamma = -200, -600$ and -800 for the $f(R)$ model



(a) (b) (c) (d)

Figure 3: The growth of (a) the energy density (b) WEC (c) SEC (d) DEC as function of red-shift z. Red, blue, and black lines correspond to $\gamma = -200, -600$ and -800. for the $f(R)$ model

tions are weak energy condition (WEC) $\rho \geq 0, \rho + p \geq 0$, strong energy condition (SEC) $\rho + 3p \geq 0, \rho + p \geq 0$, and dominant energy condition (DEC) $\rho - \mid p \mid \geq 0$. Actually, some black holes models verification dependence on this energy conditions [34]. In FIG 3. The evolution of energy conditions (b) WEC (c) SEC (d) DEC against the red shift is studied. We notice that WEC is verified over the z range . It is seen that in Fig 3. (c) the violation of SEC in low red shift values which is leading to the existence of dark energy. Moreover the DEC is violated in Fig 3. (d) that way our model at low z fits the recent observation which assume the accelerating expansion behavior of the universe.

4. THERMAL ANALYSIS

In the generalized second law of thermodynamics (GSLT), the total entropy of both the horizon and total matter inside the horizon does not decrease with time. In first law of thermodynamics, the energy is related to the entropy using $-dE = TdS$, where T is known as Hawking temperature and $S = A/4G$ is the entropy area relation. Recently, the entropy-area relation is corrected using what called the entanglement of quantum fields in and out the black hole horizon [35, 36]. Following these quantum effects which is motivated by loop quantum mechanics a type of curvature correction is obtained in the Einstein-Hilbert action that leads to the modification of both power law corrected entropy (*PLCE*) and logarithmic corrected entropy (*LCE*) [38, 39], namely:

$$S_P = \frac{A}{4G}(1 - K_\alpha A^{1-\frac{\alpha}{2}}), \tag{16}$$

where

$$K_\alpha = \frac{(4\pi)^{\frac{\alpha}{2}-1}\alpha H_o^{2-\alpha}}{(4-\alpha)r_c^{2-\alpha}}, \tag{17}$$

where $A = \pi R_X^2$ is the area of the horizon, R_X is the arbitrary horizon radius, G is Newton gravition constant, α is a postive dimensionless constant, and r_c is the cross over scale. Actually, power law correction to the entropr represented in the the second term ib the right hand side of equation (16). The curvature correction in the Einstein-Hilbert action is created due to quantum corrections into the entropy-area relationship. This leads to the logarithmic corrected entropy (*LCE*) [40]:

$$S_L = \frac{A}{4G} + \beta \log(\frac{A}{4G}) + \chi, \tag{18}$$

where β and χ are some dimensionless constants. These corrections is due to charge, mass, quantum, and thermal equilibrium fluctuations in loop quantum gravity. The system in thermal equilibrium is bounded by Hubble horizon which given by [40]:

$$R_X = \frac{1}{H}. \tag{19}$$

Now, by differentiating Eq. (16) and Eq. (18) with respect to the cosmic time and then by inserting Eqs. (5)and (6), we can reconstruct after some calculations the corrected power law $\dot{S}_\mathrm{P}$ and logarithmic entropy $\dot{S_\mathrm{L}}$ as:

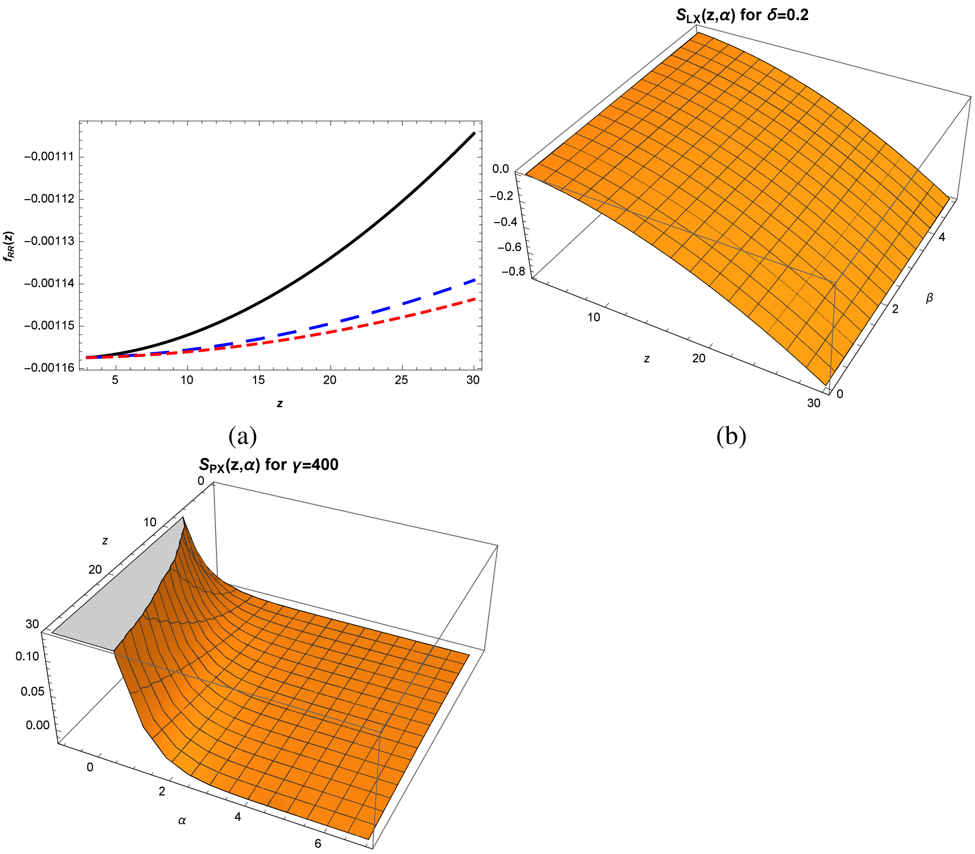
$$\dot{S}_\mathrm{P} = \frac{2\pi R_X}{R}(\dot{R_X} f_R - 6H\dot{H} f_\mathrm{RR} R_X)(1 - (4\pi)^{1-\frac{\alpha}{2}}\left(2 - \frac{\alpha}{2}\right) K\left(f_R {R_X}^2\right)^{1-\frac{\alpha}{2}}), \tag{20}$$

and

$$\dot{S_\mathrm{L}} = 2\left(\frac{\beta}{f_G R_X} + \frac{\pi R_X}{R}\right)(\dot{R_X} f_R - 6H\dot{H} f_\mathrm{RR} R_X), \tag{21}$$

which are the quantum modified version of both power law and logarithmic entropy according obtained by our model . Actually inserting f(R) gravity in *SMHDE* models plays a crucial rule in understanding the late time universe acceleration behavior, that way it my be an effective model to understand the nature of dark energy.

In FIG.4 (a) The evolution of the f_{RR} against the red shift showing f_{RR} is negative over the considered z range. In Fig.4 (b) and (c) we study the evolution of *GSLT* over the assumed red shift of (α, β) ranges. We notice that *PLCE* stays in positive level for $\alpha > 0$, while *LCE* stays in the negative level if



(a) (b) (c)

Figure 4: (a) The evolution of f_{RR} as a function of z. Red, blue, and black lines correspond to $\gamma = -200, -600$ and -800. (b) power law entropy SP and (c)logarithmic entropy SL as a function of both z and α and β for the $f(R)$ model

the arbitrary parameter $\beta > 0$. The $GSLT$ is satisfied over the whole z range if $\alpha > 0$ for the quantum power law corrected version for Hubble cut-off.

5. CONCLUSION

The growth of $f(R)$ in the framework work of Sharma - Mitall Holographic Dark Energy model ($SMHDE$) is studied, the main cosmological parameters are calculated. We reconstructed both the deceleration parameter q and the equation of state parameter ω for our model, their behavior supports the idea of accelerated expansion of the universe because $q < 0$ and $\omega > -1$ therefore quintessence mode is observed. The growth of the dark energy is studied using $\Omega^{'}$ parameter which shows a positive behavior over the whole range. The diagnostic Om is considered, we use its slope to indicate the universe mode, if slope >0 that leads to phantom mode while negative slope leads to quintessence like behavior. Our model is stable because the square speed of sound parameter is positive and this is one of the interesting feature of this model. For completeness, we have considered the energy conditions namely SEC, WEC and DEC we notice that our results supports the accelerated expansion behavior over a certain red shift range. For the thermal analysis, we studied the generalized second law of thermodynamics *GSLT* for our model using *SMHDE* universe. Both of the power law corrected entropy and logarithmic corrected entropy are considered. We notice *PLCE* is verified while *LCE* is not. The *GSLT* is hold over the considered red shift range for power law corrected scenario.

6. SUMMARY

In this work we applied f(R) modified gravity in the frame of $S-M$ Holographic Dark Energy model, the results obtained shows that our model is stable and fits the current observation of continuous expansion of universe. The thermal analysis shows that *GSLT* is verified for power law corrected case.

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